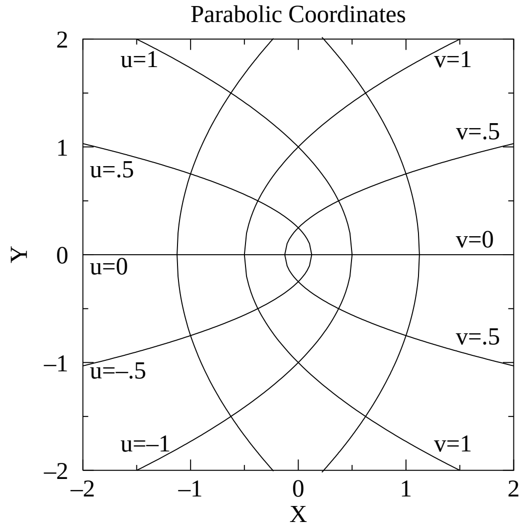
5. Consider the (2d) parabolic coordinate system (u, v):

$$
\begin{aligned}
x &= (u^2 - v^2)/2 \\
y &= uv \\
u &= \frac{y}{\sqrt{r - x}} \\
v &= \sqrt{r - x}
\end{aligned}
$$



where $r = \sqrt{x^2 + y^2}$. Find the vectors: $\mathbf{e}_u$ and $\mathbf{e}_v$. By taking derivatives of these vectors find the Christoffel symbols: $\Gamma^u_{uu}, \Gamma^u_{uv}, \Gamma^u_{vv}, \Gamma^v_{uu}, \Gamma^v_{uv}, \Gamma^v_{vv}$. Use dot products of $\mathbf{e}_u$ and $\mathbf{e}_v$ to find the metric tensor. Take derivatives of the metric tensor to confirm one of the Christoffel symbols determined above.

6. The tensor $T^{\mu\nu}$ in the usual $x^\mu = (x, y, z, ct)$ (flat) coordinate system has the following values:

$$
T^{\mu\nu} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}
$$

Find T^μ_ν and use it to produce an invariant. Consider a change of coordinates from rectangular to spherical $x^{\mu'} = (r, \theta, \phi, ct)$ where as usual:

$$
\begin{aligned}
x &= r \sin\theta \cos\phi & \qquad r &= \sqrt{x^2 + y^2 + z^2} \\
y &= r \sin\theta \sin\phi & \qquad \theta &= \cos^{-1}(z/r) \\
z &= r \cos\theta & \qquad \phi &= \tan^{-1}(y/x)
\end{aligned}
$$

Find $T^{\mu'\nu'}$, i.e., $T^{\mu\nu}$ in the spherical coordinate system.