

**Complete 5 of the following 6 problems.**

The  $S'$  frame moves with a velocity  $\beta c$  down the  $x$  axis of the  $S$  frame. The relationship between coordinates in the two frames is given by:

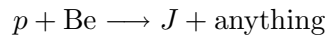
$$\text{Boost:} \quad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \text{and} \quad \begin{matrix} y' = y \\ z' = z \end{matrix}$$

$$\text{or:} \quad \mathbb{X}' = O \cdot \mathbb{X} \quad \text{where: } \mathbb{X} = (\mathbf{r}, ict)$$

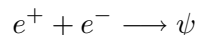
$$\text{and } O \text{ is the orthogonal matrix: } \begin{pmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

$$\text{4-vectors:} \quad \mathbf{U} = \beta c = \gamma(\mathbf{v}, ic) \quad \mathbf{P} = m_0\mathbf{U} = (\mathbf{p}, iE/c) = m_0\gamma(\mathbf{v}, ic)$$

1. In the  $S$  frame a photon with frequency  $f$  moves straight down the the  $y$ -axis. Sketch the view in the  $S'$  frame. Report the angle of incidence as seen in the  $S'$  frame. Report the photon energy in the  $S'$  frame. In the  $S$  frame another photon moves straight down the the  $x$ -axis. What is the energy of this photon as seen in the  $S'$  frame?
2. In the  $S$  frame a slightly slanting guillotine blade falls directly on the  $x$ -axis. The cut point moves up the positive  $x$  axis at a speed greater than  $c$ . As seen in a sufficiently fast moving  $S'$  frame the cut point will actually be seen to move in the negative  $x$  direction (i.e.,  $dx'/dt' < -\beta c$ ). Calculate the cut velocity in  $S'$  and find the  $\beta < 1$  needed for the reverse cut to occur. Explain what is happening in terms of lack of simultaneity. Now consider the following situation: As seen in the  $S$  frame the cut point moves up the  $x$  axis, but when the cut point hits the origin, the blade is “instantly” vaporized. The result is a cut only for  $x < 0$ . Explain what this would look like in the  $S'$  frame where the cut point moves backwards.
3. The “November Revolution” in 1974 yielded Nobels (in 1976) for Burton Richter and Sam Ting. Ting *et al.*, used the 28 GeV protons from the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL) to form a particle they called  $J$ :

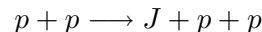


Richter *et al.*, used the  $e^+e^-$  colliding beams from SPEAR at SLAC near Stanford in California to produce an identical particle they called  $\psi$ :



Thus today we have the “ $J/\psi(1S)$ ” particle on p.84.

(a) Using the reaction:



what is the threshold energy (of the proton projectile in the lab frame) for the above  $J$  production reaction?

- (b) At SPEAR the lab frame is the cm frame. What  $e^-$  velocity (or  $\beta$ ) is needed for  $\psi$  production?
- (c) At SPEAR the  $\psi$  is produced at rest and soon decays, sometimes according to:

$$\psi \longrightarrow \pi^0 + \rho^0$$

( $\pi^0 = \text{p.26}$ ,  $\rho^0 = \text{p.28}$ ). What is the velocity (or  $\beta$ ) of the  $\pi^0$ ?

- (d) On average, how far would the  $\pi^0$  travel before it decayed?

4. Consider the tensor  $T_{\mu\nu}$  that in the  $S$  frame has the following values:

$$T_{\mu\nu} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}$$

Find  $T_{\mu'\nu'}$ , i.e.,  $T_{\mu\nu}$  in the  $S'$  frame. Report an invariant that can be formed from  $T_{\mu\nu}$  and its value.

5. Assume that  $\mathbb{D}$  is a 4-vector field with components

$$\mathbb{D} = (0, 0, \sin(5x - 3ct), i \cos(5x - 3ct))$$

Find  $\mathbb{D}'(x', ct')$ , i.e., the vector field  $\mathbb{D}$  as seen in the  $S'$  frame and expressed in terms of the  $S'$  coordinates. Calculate  $\partial_\mu \mathbb{D}_\mu$  and  $\partial'_\mu \mathbb{D}'_\mu$ . Is the result invariant?

6. The diagram below shows “synchronized” clocks in frames  $S$  and  $S'$  as viewed from the CM frame ( $S''$ ). Report how long it takes clock  $A'$  to click off two second as seen in  $S$ . SO what is the  $\gamma$  factor? Report how long it takes clock  $E$  to click off two seconds as seen in  $S'$ . As seen in  $S'$ , how far apart are  $A$  and  $E$  at the time  $t' = 3$ ? SO what is the  $\gamma$  factor? Call the distance between adjacent clocks as seen in the CM frame  $\Delta x''$ . Note that the clock  $A'$  travels a distance of  $\Delta x''$  in a time  $\Delta t' = 2$ . Use this information to write down an equation for the velocity of the  $S'$  frame relative to the  $S''$  frame ( $\gamma$  for the boost between  $S'$  and  $S''$  should enter into this equation). Note the lack of synchronization between clocks  $A'$  and  $B'$  as seen in the CM frame:  $\Delta t' = 1$  for  $\Delta x''$  separation. Write down the equation describing this lack of synchronization. Solve these two equations to show that the  $\beta$  and  $\gamma$  that connect the  $S''$  and  $S'$  frames must satisfy:  $\beta^2 \gamma^2 = \frac{1}{2}$ . Find  $\beta$ .

