Physics 366: Relativity EXAM 1 1999 February 24

Complete 5 of the following 6 problems.

The S' frame moves with a velocity βc down the x axis of the S frame. The relationship between coordinates in the two frames is given by:

$$\textbf{Boost:} \quad \begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \text{and} \quad \begin{matrix} y' = y \\ z' = z \end{matrix}$$

or: $\mathbb{X}' = O \cdot \mathbb{X}$ where: $\mathbb{X} = (\mathbf{r}, ict)$

and O is the orthogonal matrix: $\begin{pmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$

4-vectors: $\mathbb{U} = \mathbb{\beta} c = \gamma(\mathbf{v}, ic)$ $\qquad \mathbb{P} = m_0 \mathbb{U} = (\mathbf{p}, iE/c) = m_0\gamma(\mathbf{v}, ic)$

1. In the S frame a photon with frequency f moves straight down the the y-axis. Sketch the view in the S' frame. Report the angle of incidence as seen in the S' frame. Report the photon energy in the S' frame. In the S frame another photon moves straight down the the x-axis. What is the energy of this photon as seen in the S' frame?

2. In the S frame a slightly slanting guillotine blade falls directly on the x-axis. The cut point moves up the positive x axis at a speed greater than c. As seen in a sufficiently fast moving S' frame the cut point will actually be seen to move in the negative x direction (i.e., $dx'/dt' < -\beta c$). Calculate the cut velocity in S' and find the $\beta < 1$ needed for the reverse cut to occur. Explain what is happening in terms of lack of simultaneity. Now consider the following situation: As seen in the S frame the cut point moves up the x axis, but when the cut point hits the origin, the blade is "instantly" vaporized. The result is a cut only for $x < 0$. Explain what this would look like in the S' frame where the cut point moves backwards.

3. The "November Revolution" in 1974 yielded Nobels (in 1976) for Burton Richter and Sam Ting. Ting *et al.*, used the 28 GeV protons from the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory (BNL) to form a particle they called J:

$$p + \text{Be} \longrightarrow J + \text{anything}$$

Richter *et al.*, used the e^+e^- colliding beams from SPEAR at SLAC near Stanford in California to produce an identical particle they called ψ:

$$e^+ + e^- \longrightarrow \psi$$

Thus today we have the "$J/\psi(1S)$" particle on p.84.

(a) Using the reaction:

$$p + p \longrightarrow J + p + p$$

what is the threshold energy (of the proton projectile in the lab frame) for the above J production reaction?

(b) At SPEAR the lab frame is the cm frame. What e^- velocity (or β) is needed for ψ production?

(c) At SPEAR the ψ is produced at rest and soon decays, sometimes according to:

$$\psi \longrightarrow \pi^0 + \rho^0$$

($\pi^0 =$ p.26, $\rho^0 =$ p.28). What is the velocity (or β) of the π^0?

(d) On average, how far would the π^0 travel before it decayed?

4. Consider the tensor $T_{\mu\nu}$ that in the S frame has the following values:

$$T_{\mu\nu} = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix}$$

Find $T_{\mu'\nu'}$, i.e., $T_{\mu\nu}$ in the S' frame. Report an invariant that can be formed from $T_{\mu\nu}$ and its value.

5. Assume that $\mathbb{D}$ is a 4-vector field with components

$$\mathbb{D} = (0, 0, \sin(5x - 3ct), i\cos(5x - 3ct))$$

Find $\mathbb{D}'(x', ct')$, i.e., the vector field $\mathbb{D}$ as seen in the S' frame and expressed in terms of the S' coordinates. Calculate $\partial_\mu \mathbb{D}_\mu$ and $\partial'_\mu \mathbb{D}'_\mu$. Is the result invariant?

6. The diagram below shows "synchronized" clocks in frames S and S' as viewed from the CM frame (S''). Report how long it takes clock A' to click off two second as seen in S. SO what is the γ factor? Report how long it takes clock E to click off two seconds as seen in S'. As seen in S', how far apart are A and E at the time $t' = 3$? SO what is the γ factor? Call the distance between adjacent clocks as seen in the CM frame $\Delta x''$. Note that the clock A' travels a distance of $\Delta x''$ in a time $\Delta t' = 2$. Use this information to write down an equation for the velocity of the S' frame relative to the S'' frame (γ for the boost between S' and S'' should enter into this equation). Note the lack of synchronization between clocks A' and B' as seen in the CM frame: $\Delta t' = 1$ for $\Delta x''$ separation. Write down the equation describing this lack of synchronization. Solve these two equations to show that the β and γ that connect the S'' and S' frames must satisfy: $\beta^2\gamma^2 = \frac{1}{2}$. Find β.