Physics 366: Relativity EXAM 1 2026 February 16

Complete 4 of these 5 problems.

The S' frame moves with a velocity βc down the positive x axis of the S frame. The relationship between coordinates in the two frames is given by:

Boost: $$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \quad \text{and} \quad \begin{array}{c} y' = y \\ z' = z \end{array}$$

or: $\mathbb{X}' = O \cdot \mathbb{X}$ where: $\mathbb{X} = (\mathbf{r}, ict)$

and O is the orthogonal matrix: $$\begin{pmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

4-vectors: $\mathbb{U} = \mathbb{\beta} c = \gamma(\mathbf{v}, ic)$ $\quad \mathbb{P} = m_0\mathbb{U} = (\mathbf{p}, iE/c) = m_0\gamma(\mathbf{v}, ic)$

1. (**A**) Jack believes in Einstein's version of Galilean Invariance—that there will be no difference in the outcomes of experiments performed in inertial frames of differing velocities. Additionally he understands the proof that light clocks in moving frames will seem to run slow compared to those in a 'non-moving' inertial frame. But he argues that the time experienced by living things (plant growth, hair growth) is complex and need not agree with an adjacent light clock. Write a handful of sentences to convince him his concern is inconsistent with his beliefs.

   (**B**) Jill believes magnetic fields and electric fields are fundamentally different and cannot believe that they get mixed together when viewed from moving reference frames. Write a handful of sentences to convince her that something like that must be the case.

2. Assume that $\mathbb{D}$ is a 4-vector field with components

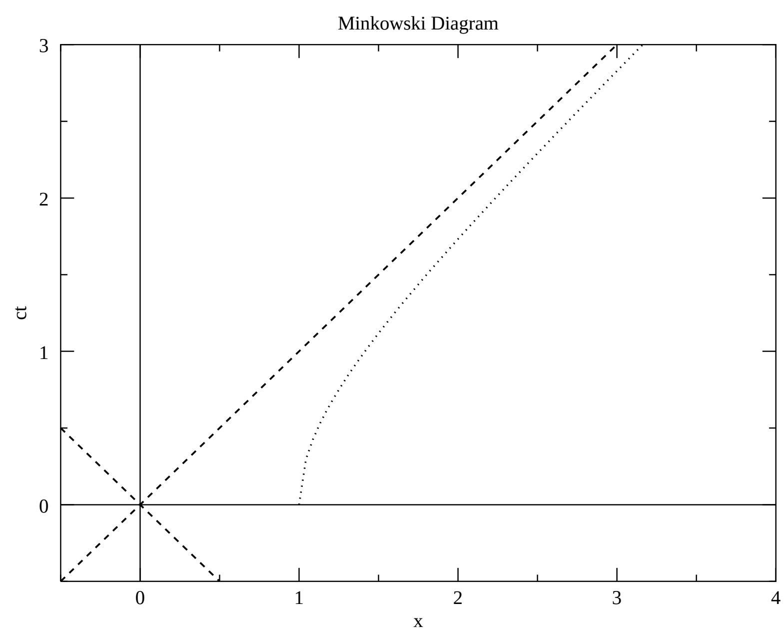
   $$\mathbb{D} = (0, 0, 0, ict)$$

   Find $\mathbb{D}'(x', ct')$, i.e., the vector field $\mathbb{D}$ as seen in the S' frame and expressed in terms of the S' coordinates. Calculate $\partial_\mu \mathbb{D}_\mu$ and $\partial'_\mu \mathbb{D}'_\mu$. Is the result invariant?

3. Consider the 4-momentum $\mathbb{P} = a(3, -2, 1, i5)$ where a is a constant. Calculate $\mathbb{P}^2$. What is the rest mass of this particle? What is γ for this particle? What is the velocity of this particle? What is the speed of the particle?

4. (Note the equations for constant acceleration were summarized in a handout) In the inertial frame shown on the attached Minkowski diagram (with units light year) a pulse of light is emitted from the origin at $t = 0$. At the same time at $x = x_0 = c^2/g \approx 1$ Ruppert starts his spacecraft with a constant acceleration g up the x axis. The dotted line shows his world line. The light, of course consistently moving at c, starts from behind Ruppert and will be catching up with Ruppert who started with initial velocity zero. Label the light's path on the Minkowski diagram **A**. Label Ruppert's path **B**. At the time $ct = 3$ label with **C** the separation between the two. Report a formula for this separation as a function of ct. For long times $ct \gg x_0$ the two will be neck-and-neck. Taylor expand your formula for the separation to first order in x_0/ct, showing that the separation gets vanishingly small.

   Inside Ruppert's spacecraft it seems as if there is a force down much like the Earth's gravitational force near the Earth's surface. On Earth if you throw a baseball straight up with an initial velocity $u_0 \ll c$ it will hit the ground at a time $2u_0/g$ after the throw. If Ruppert throws such a baseball at the same time he starts his engine ($t = 0$), since there is no actual gravitational force in the Minkowski diagram's inertial frame it will move at a constant speed. Draw the baseball's the world line on the Minkowski diagram and label it **D**. Label the point where it hits the floor of Ruppert's spacecraft **E**. To find a formula for the proper time (τ) when the baseball hits the floor, equate Rupert's location with the baseball's location. Express these locations in terms of τ. Taylor expand the hyperbolic functions ($\cosh(x) \approx 1 + x^2/2$ & $\sinh(x) \approx x$), and conclude that the same formula approximately applies for the hit time using the proper time aboard the spacecraft.



5. As shown below a uniform bar (density ρ, length L, square cross section with side $2a$) is cantilevered in the Earth's gravitational field. The stress tensor T_{ij} inside the bar is:

$$\overleftrightarrow{\mathbf{T}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_{yy} & \sigma_{yz} \\ 0 & \sigma_{yz} & \sigma_{zz} \end{pmatrix}$$

$$\text{where} \quad : \quad \sigma_{yz} = -\frac{3}{2}w(L-z)\left(1-\left(\frac{y}{a}\right)^2\right) \qquad \sigma_{zz} = \frac{3}{2}\frac{w(L-z)^2}{a^2}\,y$$

$$: \quad \sigma_{yy} = -\frac{1}{2}wy\left(1-\left(\frac{y}{a}\right)^2\right)$$

$$\text{and} \quad : \quad w = \rho g \qquad \text{cross sectional area} = A = (2a)^2$$

To understand this result, focus on the hunk to the right of the cross section located at z. While the net gravitational force on this hunk ($w(L-z)A$) may seem of primary concern, in fact the torque on this hunk about the point **P** ($w(L-z)^2A/2$) generates the larger forces basically because of the much smaller moment arm ($< a$) available at **P**. Directly on the below diagram draw little force arrows normal to the shown cross section that must counter this gravitational torque to allow a motionless bar. (There is of course no net force in the z directions, and your arrows should be consistent with that fact.) Noting that the outward normal to the hunk at the cross section is $-\widehat{\mathbf{k}}$, show that the sign of the force in the z direction calculated from the stress tensor matches the arrows you drew. Now to counter the weight the stress tensor must produce a force in the $\widehat{\mathbf{j}}$ direction on the hunk. Circle the term in the above matrix that will produce this force. Note that this force varies with y and is zero at the top and bottom of the bar. (Obviously the air cannot suck the bar up.) Explain why the below integral gives the net upward force, and show that it equals $w(L-z)A$.

$$\int_{-a}^{a} -\sigma_{yz} 2a\, dy$$