341 - class 8 - 3-1, 3-2, 3-3

3-1 : 1-d spherical solution : $\phi = \frac{A}{r} + B$; A, B constants

$$\phi_a = \frac{A}{r_a} + B$$
$$-\quad \phi_b = \frac{A}{r_b} + B$$

$$\phi_a - \phi_b = A\left(\frac{1}{r_a} - \frac{1}{r_b}\right) \Rightarrow A = \frac{\phi_a - \phi_b}{\frac{1}{r_a} - \frac{1}{r_b}}$$

$$\phi_a = \frac{\phi_a - \phi_b}{\frac{1}{r_a} - \frac{1}{r_b}} \frac{1}{r_a} + B = \frac{\phi_a - \phi_b}{1 - \frac{r_a}{r_b}} + B$$

$$\frac{\phi_a\left(1 - \frac{r_a}{r_b}\right) - \phi_a + \phi_b}{1 - \frac{r_a}{r_b}} = B$$

$$\| $$

$$\frac{\phi_b - \frac{r_a}{r_b}\phi_a}{1 - \frac{r_a}{r_b}}$$

if outside $V(\infty) = 0$ (the usual case)

then $\phi = \frac{\phi_b r_b}{r}$

3-2 1-d cylinder solution : $\phi = A \ln r + B$

$$\phi_a = A \ln r_a + B$$
$$-\quad \phi_b = A \ln r_b + B$$

$$\phi_a - \phi_b = A \ln\left(\frac{r_a}{r_b}\right)$$

$$\frac{\phi_a - \phi_b}{\ln(r_a/r_b)} = A$$

$$\phi_a = \frac{\phi_a - \phi_b}{\ln(r_a/r_b)} \ln r_a + B$$

$$\frac{\phi_a \ln(r_a/r_b) - (\phi_a - \phi_b) \ln r_a}{\ln(r_a/r_b)} = B$$

$$\| $$

$$\frac{\phi_b \ln r_a - \phi_a \ln r_b)}{\ln(r_a/r_b)} = B$$

3-3 $\quad \partial_x \nabla^2 \phi = 0$

$$\|$$

$\nabla^2 \partial_x \phi$ etc — derivatives commute

$$\partial_x \partial_y = \partial_y \partial_x$$