341 - class 7 - handout - 2.21, 2.46, dipoles

21) Note ϕ = constant except $R<r<2R$ so only there can there be nonzero E & ρ.

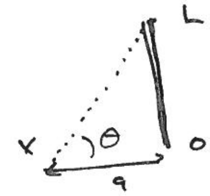
$\vec{\nabla}$ in cylindrical: $E_r = -\partial_r \phi(r) = \frac{\rho_0}{2\varepsilon_0} r$

$\nabla\cdot$ in cylindrical: $\frac{1}{r}\partial_r(rE_r) = \frac{\rho_0}{2\varepsilon_0}\frac{1}{r}\partial_r(r^2) = \frac{\rho_0}{\varepsilon_0}$

$$\frac{1}{r}\partial_r(rE_r) = \rho(r)/\varepsilon_0$$

so $\rho(r) = \begin{cases} \rho_0 & \text{between} \\ 0 & \text{if } r<R \text{ or } r>2R \end{cases}$

46) Start with potential due to rod:



rod $\phi = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda\, dy}{\sqrt{y^2+a^2}}$

Dwight 200.01

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(y+\sqrt{y^2+a^2}\right)\right]_0^L$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(\frac{L+\sqrt{L^2+a^2}}{a}\right)\right]$$

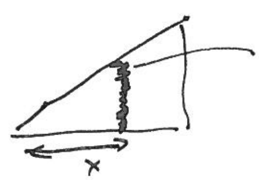
$\frac{L}{a} = \tan\theta$

$\frac{a}{\sqrt{L^2+a^2}} = \cos\theta$

$$= \frac{\frac{Q}{L}}{4\pi\varepsilon_0}\ln\left(\frac{\sin\theta+1}{\cos\theta}\right)$$

note θ will be constant

Now add up bits:



$L = \frac{b}{a}\times\frac{a}{b}$

$Q = L\,\Delta x\,\sigma$ $= \cancel{\frac{b}{a}\,x\Delta x}$

$\frac{Q}{L} = \sigma\,\Delta x$

$$\phi = \int_0^b \frac{\sigma\, dx}{4\pi\varepsilon_0}\ln\left(\frac{\sin\theta+1}{\cos\theta}\right)$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\ln\left(\frac{\sin\theta+1}{\cos\theta}\right)\int dx = \frac{\sigma b}{4\pi\varepsilon_0}\ln\left(\frac{\sin\theta+1}{\cos\theta}\right) \checkmark$$

dipoles: Consider $\vec{E}$ due to one dipole at location of other.
in cases c & d $\vec{E}$ is 2x what it is in cases a & b
in cases b & c $\vec{E}$ is in same direction & $U = -\vec{p}\cdot\vec{E}$
so: least energy c, b, a, d most energy