Class 6 - 341 - handout

2-15a with $\rho = Ar \rightarrow$ find total Q within r: $Q = \int_0^r \rho\, 4\pi r^2 dr$

$= A 4\pi \int_0^r r^3 dr$

Gauss: $\Phi = 4\pi r^2 E = \dfrac{\text{enclosed } Q}{\varepsilon_0}$

$= A\pi r^4$

Inside: $E = \dfrac{A\pi r^4}{4\pi\varepsilon_0 r^2} = \dfrac{A r^2}{4\varepsilon_0} \rightarrow \phi = \dfrac{-1}{3\cdot 4}\dfrac{A}{\varepsilon_0} r^3 + C_1$

outside $E = \dfrac{A\pi R^4}{4\pi\varepsilon_0 r^2} \longrightarrow \phi = \dfrac{A R^4}{4\varepsilon_0 r} + C_2$

require $\phi \rightarrow 0$ as $r \rightarrow \infty \Rightarrow C_2 = 0$

require $\phi(R)$ continuous: $\dfrac{A R^3}{4\varepsilon_0} = C_1 - \dfrac{A R^3}{12\varepsilon_0}$

$\dfrac{1}{3}\dfrac{A R^3}{\varepsilon_0} = C_1$

2) $r < R$: enclosed charge $= \pi r^2 l \rho$ $\quad \Phi = 2\pi r l E$

Gauss: $\Phi = \dfrac{Q}{\varepsilon_0} \rightarrow 2\pi r l E = \dfrac{\pi r^2 l \rho}{\varepsilon_0} \rightarrow E = \dfrac{\rho r}{2\varepsilon_0}$

$\phi(r=R) = 0 \Rightarrow \quad \phi = \dfrac{\rho}{4\varepsilon_0}(R^2 - r^2) \quad \rightarrow \phi = -\dfrac{1}{4}\dfrac{\rho r^2}{\varepsilon_0} + C_1$

$r > R$ enclosed charge $= \pi R^2 l \rho$ $\quad \Phi = 2\pi r l E$

Gauss $\Phi = \dfrac{Q}{\varepsilon_0} \Rightarrow 2\pi r l E = \dfrac{\pi R^2 l \rho}{\varepsilon_0} \rightarrow E = \dfrac{\rho R^2}{2\varepsilon_0 r}$

$\phi(r=R) = 0 \Rightarrow \quad \phi = \dfrac{-\rho R^2}{2\varepsilon_0}\ln\left(\dfrac{r}{R}\right) \quad \Phi = -\dfrac{\rho R^2}{2\varepsilon_0}\ln(r)$

3)
1) enclosed charge $\propto R^2 \rightarrow$ C
2) enclosed charge does not change with $R \rightarrow$ A
3) same thing for sphere $\rightarrow$ A
4) enclosed charge $\propto r^2$; Area $\propto r \Rightarrow E \propto r \rightarrow$ B
5) $\Phi \propto$ charge $\propto r^2 \rightarrow$ C
6) same enclosed charge, same flux $\rightarrow$ A
7) same for sphere $\rightarrow$ A
8) $E \propto \frac{1}{r} \rightarrow$ D
9) D
10) D $\quad \vec{E} \perp \hat{n}$