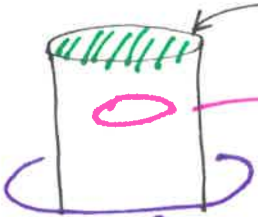
341 – class 26 – old exam 4 & 5

4)



current density $J = \frac{I}{\pi R^2}$

inside loop: enclosed current $= \pi r^2 J = I \frac{r^2}{R^2}$

$\oint \vec{B} \cdot \vec{dl} = B\, 2\pi r = \mu_0 I \frac{r^2}{R^2}$

$B = \frac{\mu_0 I r}{2\pi R^2}$

outside loop
all current enclosed!

$\oint B \cdot dl = \mu_0 I = 2\pi r B \Rightarrow B = \frac{\mu_0 I}{2\pi r}$

5) wlog I'd take $\vec{r} = (d, 0, 0)$; fully general $\vec{r} = (x, y, 0)$ where $\sqrt{x^2+y^2} = d$

↳ rotational symmetry says all points a distance d from center are equivalent

$\vec{r}' = R(\cos\theta, \sin\theta, 0)$

$\vec{dl} = d\vec{r}' = R\,d\theta\,(-\sin\theta, \cos\theta, 0)$

$\vec{r} - \vec{r}' = (d - R\cos\theta, -R\sin\theta, 0) \rightarrow |r - r'| = \sqrt{R^2 + d^2 - 2Rd\cos\theta}$

$$\vec{dl} \times (\vec{r} - \vec{r}') = \begin{vmatrix} i & j & k \\ -\sin\theta & \cos\theta & 0 \\ d - R\cos\theta & -R\sin\theta & 0 \end{vmatrix} R\,d\theta = \overset{R\,d\theta}{\hat{}}\,\left(0, 0, \underset{\text{just in } z \text{ direction}}{R\sin^2\theta - d\cos\theta + R\cos^2\theta}\right)$$

$$B_z = \frac{\mu_0 I}{4\pi} \int_{-\pi}^{\pi} \frac{(R - d\cos\theta)\, R\,d\theta}{(R^2 + d^2 - 2Rd\cos\theta)^{3/2}}$$