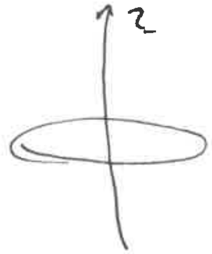
341 - class 25 - B2.pdf #2 & 3

#2



$$B_z = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$$

$$\int_{-\infty}^{\infty} B_z\,dz = \frac{\mu_0 I R^2}{2}\int_{-\infty}^{\infty}\frac{dz}{(z^2+R^2)^{3/2}} = \frac{\mu_0 I R^2}{2}\left[\frac{1}{R^2}\frac{z}{\sqrt{z^2+R^2}}\right]_{-\infty}^{\infty}$$

$$= \frac{\mu_0 I R^2}{2}\frac{1}{R^2}\left[\frac{\infty}{\infty} - \frac{-\infty}{\infty}\right] = \mu_0 I \checkmark$$

#3: (a) $\nabla\cdot A = B\,\partial_y x = 0$ $\quad \nabla\times A = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 0 & Bx & 0 \end{vmatrix} = B\hat{k}$

(b) $A = \frac{1}{2}\begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ x & y & z \end{vmatrix} = \frac{1}{2}(-y, x, 0)$ $\quad \nabla\cdot A = \frac{1}{2}\left[-\partial_x y + \partial_y x\right] = 0$

$$\nabla\times A = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ -y & x & 0 \end{vmatrix} = \frac{1}{2}(1+1)\hat{k} = \hat{k}$$

alt:  $\hat{k}\times\hat{r} = r\sin\theta$ = cylinder r direction: $\hat{\phi}$

Book: $\nabla\times F = \hat{r}\left(\frac{1}{r}\partial_\phi F_z - \partial_z F_\phi\right) + \hat{\phi}\left(\frac{\partial F_r}{\partial z} - \partial_r F_z\right) + \hat{k}\frac{1}{r}\left(\partial_r(rF_\phi) - \frac{\partial F_r}{\partial \phi}\right)$

not: A_z & $A_r = 0$; $\partial_z A_\phi = 0$ but $\frac{1}{r}\partial_r(r A_\phi)$

$= \frac{1}{r}\partial_r\left(\frac{1}{2}r^2\right) = 1$

$\Rightarrow \nabla\times A = \hat{k}$

(c) only A_z is non zero; only $\partial_r A_z$ non zero

$\nabla\times A = -\hat{\phi}\,\partial_r A_z = \frac{\mu_0 I}{2\pi}\frac{1}{r}$

(d) only A_ϕ is non zero; only $\partial_r A_\phi$ is non zero

$\nabla\times A = \hat{k}\frac{1}{r}\partial_r(r A_\phi) = \hat{k}\frac{1}{r}\partial_r\left(\frac{\mu_0 N I}{2}r^2\right) = \mu_0 N I\,\hat{k}$

↳ if $A_\phi \propto \frac{1}{r}$ is zero $\quad \partial_r \frac{1}{r} = \frac{-1}{r^2}$

(e) A_ϕ depends on θ & r; rest zero

$$\nabla\times A = \hat{r}\frac{1}{r\sin\theta}\underbrace{\partial_\theta(A_\phi\sin\theta)}_{\partial_\theta \sin^2\theta = 2\sin\theta\cos\theta} - \hat{\theta}\frac{1}{r}\partial_r(r A_\phi)$$

(e) continued: overall factor $\frac{\mu_0 m}{4\pi}$

$$\nabla \times A = \frac{\mu_0 m}{4\pi}\left[\hat{r}\,\frac{2\cos\theta}{r^3} + \hat{\theta}\,\frac{\sin\theta}{r^3}\right]$$

we can "simplify" this by noting $\hat{k} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{3\cos\theta\,\hat{r} - \hat{k}}{r^3}\right]$$

← this is a dipole field.