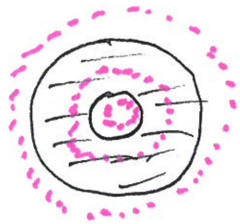
341 – class 15 – handout + 4-18

1)



Consider Gaussian surfaces as shown: inside $(r<a)$, in $(a<r<b)$, outside $(r>b)$

Inside: no enclosed free charge

$D\,2\pi r l = 0$

$D = 0$

$E = 0$

$\phi = 0 \; : \; 0 \le r < a$

In:

$D\,2\pi r l = \lambda l$

$D = \frac{\lambda}{2\pi r}$

$\varepsilon E = \varepsilon_0 K E$

$E = \frac{\lambda}{2\pi \varepsilon_0 K r}$

$\underset{0}{\phi(a)} - \phi(r) = \int_a^r E\,dr$

$-\phi(r) = \int_a^r \frac{\lambda}{2\pi\varepsilon_0 K r} = \frac{\lambda}{2\pi\varepsilon_0 K} \ln(r/a)$

$\phi(r) = -\frac{\lambda}{2\pi\varepsilon_0 K} \ln(r/a)$

Outside: no enclosed free charge

$D\,2\pi r l = 0$

$D = 0$

$E = 0$

$\phi = \text{constant} = \frac{-\lambda}{2\pi\varepsilon_0 K} \ln\left(\frac{b}{a}\right)$

2) Exactly as above consider 3 Gaussian surfaces except now <u>spheres</u> not cylinders. Every surface encloses exactly the free charge Q so

$4\pi r^2 D = Q$

$D = \frac{Q}{4\pi r^2} \longrightarrow E = \frac{Q}{4\pi K \varepsilon_0 r^2}$

use $K=1$ if not in dielectric otherwise use K of the dielectric

b) $\rho_b = -\vec{\nabla}\cdot\vec{P} = -\varepsilon_0 \chi \vec{\nabla}\cdot\vec{E}$

$= -\frac{\chi}{K}\vec{\nabla}\cdot\vec{D} = 0$ (usual case)

$\sigma_b = \hat{n}\cdot\vec{P} \rightarrow$ @ $r=b$ $\hat{n}=\hat{r} \rightarrow \sigma_b = +\varepsilon_0\chi \frac{Q}{4\pi K \varepsilon_0 b^2}$

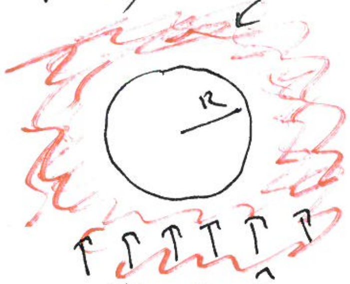
$\varepsilon_0 \chi E = \frac{\chi}{K} D$

$= \frac{\chi}{K}\frac{Q}{4\pi b^2}$

@ $r=a$, $\hat{n} = -\hat{r}$ $\qquad \sigma_b = \frac{-\chi}{K}\frac{Q}{4\pi a^2}$

total bound $= \sigma_b\big|_{r=a} 4\pi a^2 + \sigma_b\big|_{r=b} 4\pi b^2 = (-Q+Q)\frac{\chi}{K} = 0$

4-18) dielectric K



$\vec{E} = E_0 \hat{k}$

$\phi = -E_0 z = -E_0 r P_1$

As usual:

$\phi_{in} = \sum A_n r^n P_n(\cos\theta)$ [cuz C_n diverge at $r=0$]

$\phi_{out} = \underbrace{-E_0 r P_1}_{\text{makes uniform } E \text{ at } r=\infty} + \sum C_n/_{r^{n+1}} P_n(\cos\theta)$

$\phi_{in}\Big|_{r=R} = \phi_{out}\Big|_{r=R} \Rightarrow A_n R^n = \dfrac{C_n}{R^{n+1}}$ (for $n \neq 1$)

$A_1 R^1 = -E_0 R + \dfrac{C_1}{R^2}$

rewrite

$A_1 = -E_0 + \dfrac{C_1}{R^3}$

$D_{r\,in}\Big|_{r=R} = D_{r\,out}\Big|_{r=R}$ [cuz no free charge at $r=R$]

$\quad \parallel \qquad\qquad \parallel$

$-\varepsilon_0 \partial_r \phi_{in} \qquad -K\varepsilon_0 \partial_r \phi_{out}$ (mult by -1 & divide by ε_0)

$n A_n R^{n-1} = -(n+1) K \dfrac{C_n}{R^{n+2}}$ (for $n \neq 1$)

$A_1 = \left(-E_0 - 2\dfrac{C_1}{R^3}\right) K$

for any $n \neq 1$ we have 2 homo linear eqs for A_n & C_n

$\Rightarrow A_n = C_n = 0$

for $n=1$ $\qquad A_1 = -E_0 + \dfrac{C_1}{R^3}$ $\leftarrow$ mult by $2K$ & add

$A_1 = \left(-E_0 - \dfrac{2C_1}{R^3}\right) K$

$(2K+1) A_1 = -3KE_0 \rightarrow A_1 = \dfrac{-3KE_0}{2K+1}$

$\phi_{in} = A_1 r P_1 = A_1 z = \dfrac{-3KE_0}{2K+1} z$

$\vec{E}_{in} = -\nabla\phi = \dfrac{3K}{2K+1} E_0 \hat{k}$ ✓