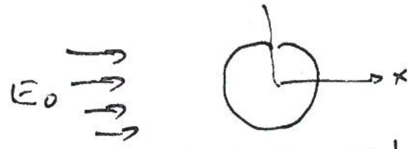
Class 14 - 341 - 4.10 & 4.15

10) Laplace - $\phi_{out} = -E_0 r\cos\theta + \sum C_n r^{-n}\cos n\theta$

$\phi_{in} = \sum A_n r^n \cos(n\theta)$



even in θ! (no sin)

No free charge on surface $\Rightarrow D_r^+ = D_r^-$

so $\partial_r \phi_{out} = K\,\partial_r \phi_{in}$

$\phi_{in}|_R = \phi_{out}|_R$ ← voltage continuous

$$\left(-E_0 R + \frac{C_1}{R}\right)\cos\theta + \sum_{n=2}^{\infty} C_n R^{-n}\cos n\theta = \sum A_n R^n \cos(n\theta)$$

so: $-E_0 R + \frac{C_1}{R} = A_1 R$ & $C_n R^{-n} = A_n R^n$

$$\partial_r \phi_{out}\Big|_R = \left(-E_0 - \frac{C_1}{R^2}\right)\cos\theta + \sum_{n=2} \frac{-nC_n}{R^{n+1}}\cos n\theta$$

$$= K\,\partial_r\phi_{in} = K\sum n A_n R^{n-1}\cos n\theta$$

So: $-E_0 - \frac{C_1}{R^2} = K A_1$ & $-\frac{nC_n}{R^{n+1}} = K n A_n R^{n-1}$

For the red eqs show $C_n = A_n = 0$ is a solution hence the solution

[if matrix of coef's of linear eq has det $\neq 0$ the sol'n to homogeneous linear Eqs is zero]

repeat top green eq / R

$$\left.\begin{aligned} -E_0 &= \frac{A_1}{R^2}... \end{aligned}\right.$$

$-E_0 = \frac{A_1}{R^2} - \frac{C_1}{R^2}$ } 2 eq

$-E_0 = KA_1 + \frac{C_1}{R^2}$ } 2 unknown

$-2E_0 = (K+1)A_1$

$\frac{-2E_0}{K+1} = A_1$

$\Rightarrow E_{inside} = -\nabla\phi = \frac{2}{K+1}E_0\,\hat{x}$

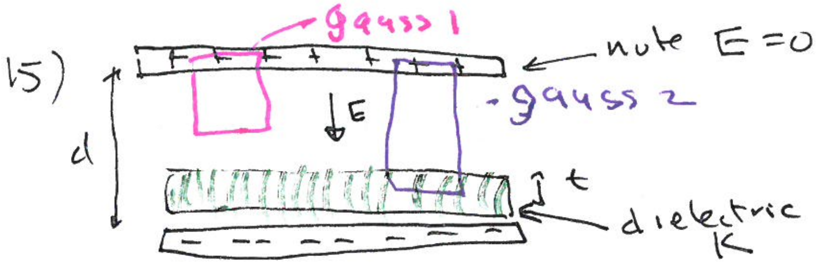
mult top by K & subtract

$(K-1)E_0 = (K+1)\frac{C_1}{R^2}$

$\frac{(K-1)}{(K+1)}E_0 R^2 = \underline{C_1}$

remark: terms like $\ln(r)$ are ruled out as no net charge on cylinder

15)



note $E = 0$ inside conductor

$D = \sigma_f$ everywhere between plates

Gauss 1: flux $= E_1 A$ where A is top/bottom area of gauss

enclosed charge is all free; original gauss $\Rightarrow EA = \frac{\sigma_f A}{\varepsilon_0}$

$E_1 = \frac{\sigma_f}{\varepsilon_0}$ as usual

Gauss 2. using D_2: $\phi = \underbrace{D_2}_{K\varepsilon_0 E_2} \cdot A = \sigma_f A$ (no count bound charge)

$$E_2 = \frac{\sigma_f}{K\varepsilon_0}$$

Now $\Delta V = E_2 \cdot t + E_1 (d - t)$

$$= \frac{\sigma_f}{\varepsilon_0}\left(\frac{t}{K} + d - t\right)$$

$$\frac{\Delta V}{\left(\frac{t}{K} + d - t\right)} = \frac{\sigma_f}{\varepsilon_0}$$

$$E_1 = \frac{\Delta V}{\left(\frac{t}{K} + d - t\right)}$$

$$E_2 = \frac{1}{K}\,\frac{\Delta V}{\left(\frac{t}{K} + d - t\right)}$$