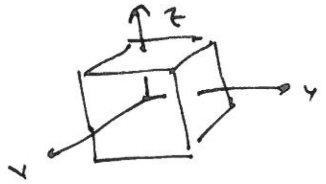
Class 13 – 341 – 4·2, 4·3

2) 

faces of cube at $x, y, z = \pm a$

$\rho_b = -\nabla\cdot P = -3A \Rightarrow$ volume $= (2a)^3$

total volume charge $= -3A(2a)^3$

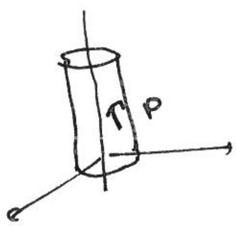
Consider the face at $x = +a$ with $\hat{n} = \hat{i}$; $\vec{P}\cdot\hat{n} = Aa$ ← constant

$\int P\cdot n \, dA$ for that face $= Aa(2a)^2$

Consider the face at $x = -a$ with $\hat{n} = -\hat{i}$ $\vec{P}\cdot\hat{n} = +Aa$

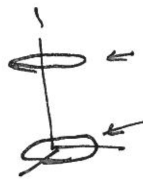
as above $\int P\cdot n\, dA = Aa(2a)^2$

6 faces $\Rightarrow$ $6 \times Aa(2a)^2 = 3A(2a)^3$ ... total zero

3) 

$\nabla\cdot P = 0$ inside rod

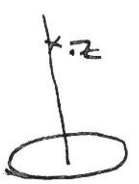
$P\cdot n = +P$ on top ; $-P$ on bottom

equivalent problem: 

← $\sigma = +P$

← $\sigma = -P$

we've worked the problem of E directly above a charged disk: but derive it again below via ϕ



$$\phi = \frac{1}{4\pi\epsilon_0}\int_0^R \frac{\sigma\, 2\pi r\, dr}{\sqrt{z^2+r^2}} = \frac{\sigma}{4\epsilon_0}\int_0^{R^2}\frac{du}{\sqrt{z^2+u}} \qquad r^2 = u$$

$$= \frac{\sigma}{4\epsilon_0}\left[\frac{(z^2+u)^{1/2}}{1/2}\right]_0^{R^2} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{z^2+R^2} - z\right]$$

$$E_z = -\partial_z\phi = -\frac{\sigma}{2\epsilon_0}\partial_z\left[\sqrt{z^2+R^2} - z\right] = -\frac{\sigma}{2\epsilon_0}\left[\frac{z}{\sqrt{z^2+R^2}} - 1\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

E field is just sum of 2 such terms $\sigma = P$ top, $\sigma = -P$ bottom

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{(z-L)}{\sqrt{(z-L)^2+R^2}} - 1 + \frac{(z)}{\sqrt{z^2+R^2}}\right]$$

FYI: expect as $z\to\infty$ $E = \dfrac{P\pi R^2 L}{4\pi\epsilon_0}\dfrac{2}{z^3}$ & mathematica confirms