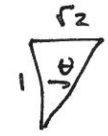
1) $(x,y,z) = (1,1,1) \longrightarrow (r, z, \phi) = (\sqrt{2}, 1, 45^\circ)$

$\longrightarrow (r, \theta, \phi) = (\sqrt{3}, \quad , 45^\circ)$



$\theta = \tan^{-1}\left(\frac{\sqrt{2}}{1}\right) = 54.7^\circ$

or

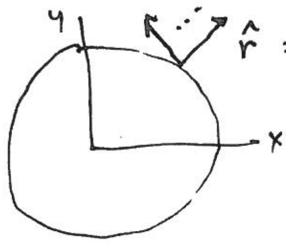
$z = r\cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{z}{r}\right)$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7^\circ$

2) $\vec{A} = (xy, 2yz, 3zx)\Big|_{1,1,1}$
$= (1, 2, 3)$

in cylinder $\hat{e}_\theta = (-\sin\theta, \cos\theta)$

$\hat{r} = (\cos\theta, \sin\theta)$



$\vec{C} = \frac{1}{r}(\cos\theta, \sin\theta, 0) + r(-\sin\theta, \cos\theta, 0)$

$= \left(\frac{1}{r}\cos\theta - r\sin\theta, \frac{1}{r}\sin\theta + r\cos\theta, 0\right)\Big|_{\sqrt{2}, 1, 45^\circ}$

$= \left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} - \sqrt{2}\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} + \sqrt{2}\frac{1}{\sqrt{2}}, 0\right)$

$= \left(\frac{1}{2} - 1, 1 + \frac{1}{2}, 0\right) = \left(-\frac{1}{2}, \frac{3}{2}, 0\right)$

$\vec{B} = \frac{\sin\theta}{r}\hat{\theta} = \frac{\sin\theta}{r}\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}\right) = \frac{\sqrt{2/3}}{\sqrt{3}}\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}\right)$

$= \left(\frac{1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}, -\frac{2}{3\sqrt{3}}\right)$

$\hat{\theta} = \hat{\phi} \times \hat{r} = \begin{vmatrix} i & j & k \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{vmatrix}$

$= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)$

$= \left(\frac{1}{\sqrt{3}}\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\frac{1}{\sqrt{2}}, -\frac{\sqrt{2}}{\sqrt{3}}\right)$

$= \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}\right)$

3): $\begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ xy & 2yz & 3zx \end{vmatrix} = (-2y, -3z, -x)$

B: only $F_\theta \neq 0$ $\quad \nabla \times B = \hat{e}_\phi \frac{1}{r}\frac{\partial(rF_\theta)}{\partial r} = 0 \quad \leftarrow \frac{r\sin\theta}{r} \Rightarrow 0$

C: $F_r = \frac{1}{r}$ $\quad F_\theta = r$ $\quad F_z = 0$

$\nabla \times C = \hat{k}\frac{1}{r}\frac{\partial}{\partial r}(rF_\theta) = \hat{k}\frac{1}{r}\frac{\partial}{\partial r}(r^2) = \hat{k}\, 2$

4) $\bar{\nabla} \cdot \vec{A} = y + 2z + 3y$

$\nabla \cdot B = \frac{1}{r\sin\theta} \frac{\partial}{\partial\theta}\left(\frac{\sin^2\theta}{r}\right) = \frac{2}{r^2} \cos\theta$

$B_r$ & $B_\phi = 0$

$B_\theta = \frac{\sin\theta}{r}$

$\nabla \cdot C = \hat{k} \frac{1}{r} \partial_r (r C_\phi) = \hat{k} \frac{1}{r} \partial_r (r^2) = 2$

$C_z = 0$

$C_r = \frac{1}{r}$

$C_\phi = r$