Homework 4: Consider an infinite cylindrical shell of radius $R = 1$, coaxial with the z axis. The voltage on the surface of the shell is given by:

$$V(\theta) = \begin{cases} +1 & \text{for } |\theta| < \pi/4 \\ -1 & \text{for } |\theta - \pi| < \pi/4 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

Using 'Fourier's Trick', find the series solution to Laplace's equation inside this cylinder. (Hint: Is this $V(\theta)$ even or odd?)

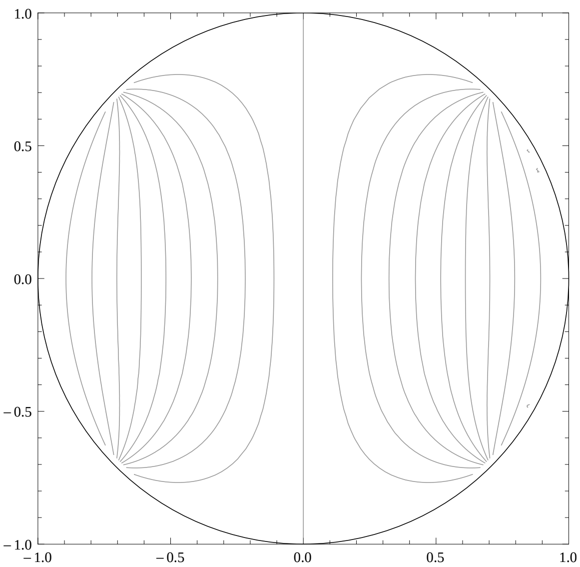
A nice feature of this problem is that Laplace's equation can be solved exactly:

$$\phi(x, y) = \frac{1}{\pi}\left[\arctan\left(\frac{\sqrt{2}\,(x+y)}{1-r^2}\right) + \arctan\left(\frac{\sqrt{2}\,(x-y)}{1-r^2}\right)\right] \tag{2}$$

```
D[D[phi[x,y],x],x]+D[D[phi[x,y],y],y]
Simplify[%]
Out[6]= 0
```

which allows you to compare truncated versions of your infinite series to the exact result. Lets pick the point $\mathbf{p} = (x, y) = (.5, 0)$ as a typical point and compare results. Calculate $\phi(\mathbf{p})$ when your sum is truncated to one, two, three, ..., six non-zero terms. Display these results along with the exact result. An easy way to do this is to include the sum-limit in the function definition:

```
phi[x_,y_]=(ArcTan[Sqrt[2](x+y)/(1-(x^2+y^2))]+ArcTan[Sqrt[2](x-y)/(1-(x^2+y^2))])/Pi
phi2[r_,t_,m_]=Sum[A[[n]]/Pi r^n Cos[n t],{n,1,2 m+1}]
Table[phi2[.5,0,m],{m,0,5}]
Table[phi2[.5,0,m]-phi[.5,0],{m,0,5}]
```



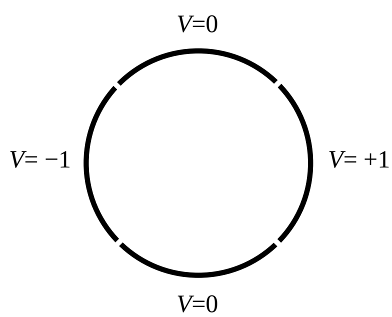


Figure 1: A contour plot of the exact solution (left). A display showing the voltages on the surface of the cylinder (right).