1. Consider a semi-infinite line charge that is coincident with the positive z axis, starting at $z = a$ and "ending" at $z = +\infty$. The charge per length $\lambda(z)$ depends on position:

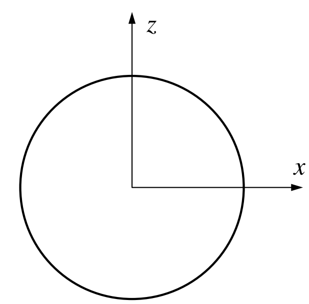
$$\lambda(z) = \frac{\Lambda}{\sqrt{z-a}}$$

where Λ is a constant. What is $\mathbf{r}'$? Set up the integral giving the electric field vector at a point: $\mathbf{r} = x\hat{\mathbf{i}} + z\hat{\mathbf{k}} = (x, 0, z)$. Do not actually calculate the integral.

2. In spherical coordinates, the azimuthally-symmetric voltage on a spherical shell (origin-centered; radius R) is given by:

$$V(\theta) = 4\cos^2\theta - 1 = 4c^2 - 1$$

where $c = \cos\theta$.



(a) Using the provided circle that represents the pole-to-pole great circle that is the intersection of the xz plane and the sphere, mark where the voltage is positive (mark +), negative (mark −) or zero (mark 0). Would you call this an even, odd or neither voltage distribution? Explain.

(b) Write down the sequence of steps and words (Fourier's Trick) to convince me that the voltage inside the sphere is given by:

$$\phi(r,\theta) = \sum_{n=0}^{\infty} A_n r^n \, P_n(\cos\theta) \tag{1}$$

where the constants A_n can be calculated from:

$$A_n = \frac{\int_0^\pi (4\cos^2\theta - 1)\, P_n(\cos(\theta))\, d\theta}{R^n \frac{2}{2n+1}} \tag{2}$$

Look carefully at the above: this answer is actually slightly wrong (a missing detail)! What's the problem? For what values of n is A_n automatically zero just based on symmetry?

(c) The Mathematica command:

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Table[Integrate[ (4 x^2-1) LegendreP[n,x],{x,-1,1}], {n,0,5}]
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which produces output:

$$\left\{\frac{2}{3}, 0, \frac{16}{15}, 0, 0, 0\right\}$$

and in fact all the integrals for $n > 2$ are zero. Write down the values of A_n for $n = 0, 2$, and put them together to find the exact expression for ϕ (as in Eq. (1) above).

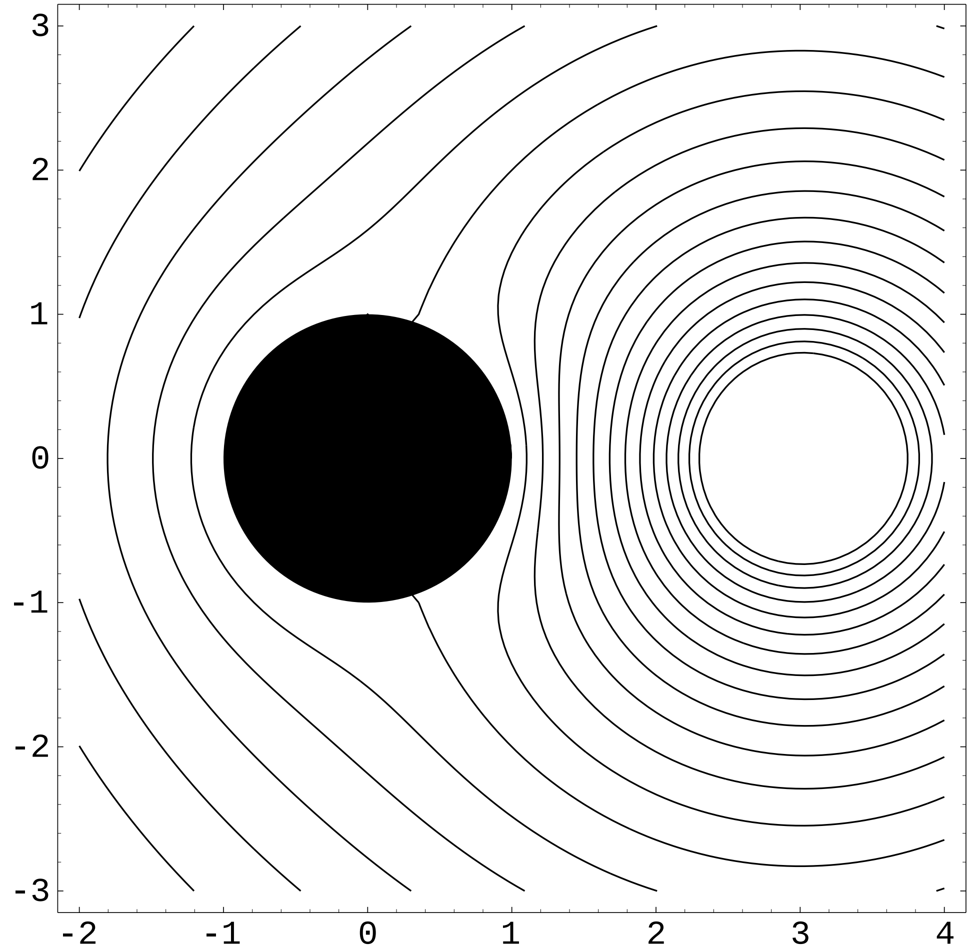
3. Consider an infinite uniform line charge ($\lambda > 0$) parallel to the z axis and piercing the xy plane at $(x, y) = (3, 0)$ next to an *uncharged* infinite conducting cylinder of radius 1, also parallel to the z axis but with center at $(x, y) = (0, 0)$. The exact solution for the voltage is displayed on the following page, with isopotential contours: $\{-3.2, -3, -2.8, -2.6, -2.4, \ -2\ln(3) \approx -2.2, -2, -1.8, \ldots, -.2, 0, .2, .4\}$. The big black circle is the conducting cylinder all at a voltage of $-2\ln(3)$. Below that contour plot and aligned with it is a plot of the voltage along the x axis: $\phi(x, 0)$. Note that the voltage is constant where the cylinder is and approaches infinity as the charged wire is approached.

   (a) On the contour plot draw in lots of $\vec{\mathbf{E}}$-field lines (include arrows) paying particular attention to E lines that touch or surround the conducting cylinder. Do any lines start or stop on the conducting cylinder? Mark with **A** a spot where the electric field is particularly small; mark with **B** a spot where the electric field is particularly big.

   (b) Consider how the electrons in the conducting cylinder have responded to the electric field of the wire. Show the location of charge on the surface of the conductor. On the below contour diagram, denote the location of positive charges with little "+" and negative charges with "–". Relate (words required) the sign and magnitude of the cylinder's surface charge σ to the electric field just outside of the conducting cylinder. Where on the surface of the cylinder is $|\sigma|$ the greatest? Where is it zero?

   (c) Consider a Gaussian cylinder of length ℓ, coaxial to and hugging the conducting cylinder. Report the net electric flux through that Gaussian cylinder. Consider a small-radius (e.g., $r = .1$) Gaussian cylinder (again with axis parallel to the z axis). Mark with **C** where such a cylinder could be centered and have a positive net flux; mark with **D** where such a cylinder could be centered and have a negative net flux.

   (d) Draw an arrow showing the direction of the net force on the cylinder? Explain!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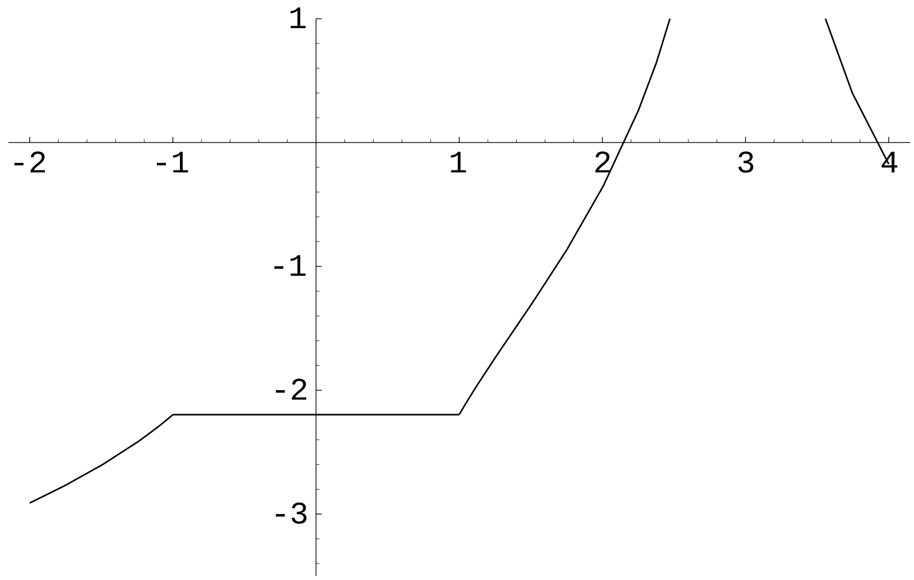
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4. Consider two conducting coaxial infinite cylindrical shells surrounded by vacuum. The inner cylinder (of radius a) carries a surface charge density $+\sigma$; the outer (of radius b) carries the opposite surface charge density $-\sigma$. (Note: σ being the same does not imply the total charges are the same.)

   (a) Use Gauss' Law to find the electric field in the three regions: (i) $r < a$, (ii) $a < r < b$, and (iii) $b < r$.

   (b) Integrate $\vec{\mathbf{E}}$ to find the electric potential difference between the inner and outer shell.