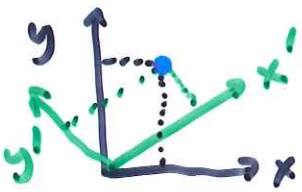


Note the following about vectors = tensors

→ The coordinates assigned to a physical point depends on the frame

→ We can calculate the coordinates in a new frame using a matrix and the coordinates in old frame

→ Some quantities (mass, time, temperature, ...) are invariant. Dot products are invariant

→ The formula for dot product: $A_x B_x + A_y B_y + A_z B_z$ has same form in any frame

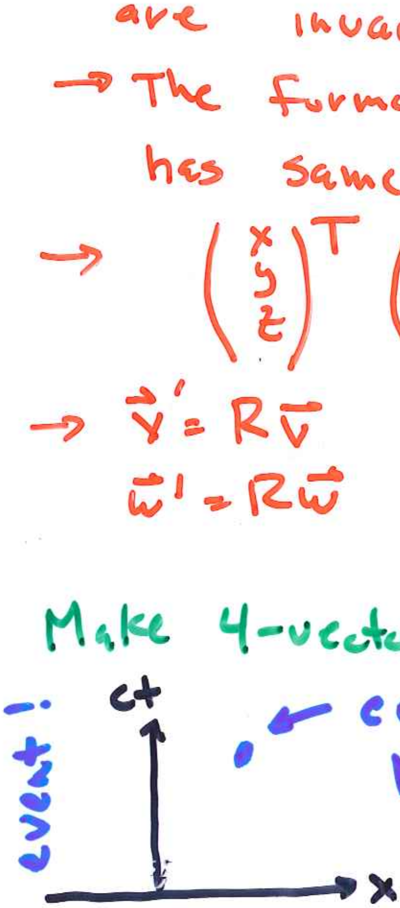
→ $\begin{pmatrix} x \\ y \\ z \end{pmatrix}^T \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$ matrix mult
T = transpose

→ $\vec{v}\,' = R\vec{v}$
$\vec{w}\,' = R\vec{w}$

$\vec{v}\,' \cdot \vec{w}\,' = (Rw)^T (Rv) = w \underbrace{R^T R}_{1} v$

$R^{-1} = R^T$ orthogonal matrix

Make 4-vector $= (\vec{r}, ict)$ with invariant $r^2 - c^2 t^2$



← coordinates depend on frame
Minkowski diagram allows us to read those coordinates directly. Use Minkowski to explain:

Lorentz contraction: S see's S' stuff contracted **and** S' sees S stuff contracted $L = \frac{L_0}{\gamma}$

Time dilation: S see's S' clocks run slow **and** S' sees S clocks run slow $t = t_0 \gamma$

Important point: "Now" & "at the same time" depends on frame

$\vec{r}$

$y = r_2$

$\mathbb{X} = (\vec{r}, ict)$

$X_2 = y$

X_μ

$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$

$v = \sqrt{v \cdot v}$

$$d\tau = \frac{\Delta X}{} \quad d\tau = \underbrace{\sqrt{\Delta t^2 - \frac{1}{c^2}\Delta \vec{r}^{\,2}}}$$

proper time.

$r^2 - c^2 t^2$ — invariant

$$= \Delta t \sqrt{1 - \frac{1}{c^2}\frac{\Delta r^2}{\Delta t^2}}$$

$$= \frac{\Delta t}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \frac{v^2}{c^2} = \beta^2$$

$$U = \frac{d\mathbb{X}}{d\tau} = \gamma \frac{d}{dt}(\vec{r}, ict)$$

$$= \gamma(\vec{v}, ic)$$

$$\gamma^2\left[v^2 - c^2\right] = -c^2\gamma^2\left[1 - \frac{v^2}{c^2}\right]$$

$$= -c^2$$

$\vec{L} = r \times P$

$\tau = r \times F$

$$U^2 = -c^2$$

$$\frac{d}{d\tau} U^2 = 2\, U \cdot A = 0$$

$E_i E_i$

$$\vec{\nabla}\phi \longrightarrow \left(\vec{\nabla}, \frac{\partial}{\partial ict}\right)$$

$$\left(\vec{\nabla}, -\frac{i}{c}\partial_t\right) \quad \square$$

$M_{ij} \longrightarrow M'_{k\ell} = R_{ki} M_{ij} R_{\ell j}$

$r'_i = M_{ij} r_j$

$E_i E_j$

rotation

$[R^T]_{j\ell}$

$R_{ki} M_{ij} [R^T]_{j\ell}$

L_3

$R M R^T = \underbrace{R M R^{-1}}$

$r_i P_j - r_j P_i = \begin{bmatrix} \cdot & 0 & \\ & \cdot & \\ & & \cdot \end{bmatrix}^T$

$R_{ijk} = R_{i\ell\ell}$

$r_i r_i = |r|^2$

$X = (\vec{r}, ict)$

$A = (\vec{A}, i\phi/c)$

$J = (\vec{J}, i\rho c)$

$\nabla \cdot J + \partial_t \rho = 0$

$\square \cdot J = \left(\vec{\nabla} \cdot \vec{J} + \frac{\partial}{\partial ict} i\rho c\right)$

$\partial_t \vec{A} + \frac{1}{c^2} \partial_t \phi = 0$ Lorentz Gau

$\square \cdot A = 0$

$B_3 = \partial_1 A_2 - \partial_2 A_1$

E, B

$$\partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$$

$\partial_4 = \frac{\partial}{\partial ict}$

$E_1 = -(\partial_1 \phi + \partial_t A)$

$ic\partial_4 =$

$A_4 \frac{c}{i}$

$= -ic(-\partial_1 A_4 + \partial_4 A_1)$

$\frac{iE_1}{c}$

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$$\partial_\nu F_{\mu\nu} = \begin{array}{cc} \left[\begin{array}{ccc|c} 0 & B_3 & -B_2 & \\ & 0 & B_1 & \\ & & 0 & \\ \hline & \frac{i}{c}\vec{E} & & 0 \end{array}\right] & \leftarrow -\frac{i}{c}\vec{E} \end{array}$$

$$\nabla \cdot B = \square$$

$$\epsilon_{ijk} B_k$$

$$\nabla \times E + \partial_t B = \square$$

$$\nabla \times B = \mu_0 \varepsilon_0 \partial_t E \ldots = \mu_0 J$$

$$\boxed{\partial_\nu F_{\mu\nu} = \mu_0 J}$$

$$\vec{F} = q(E + v \times B)$$

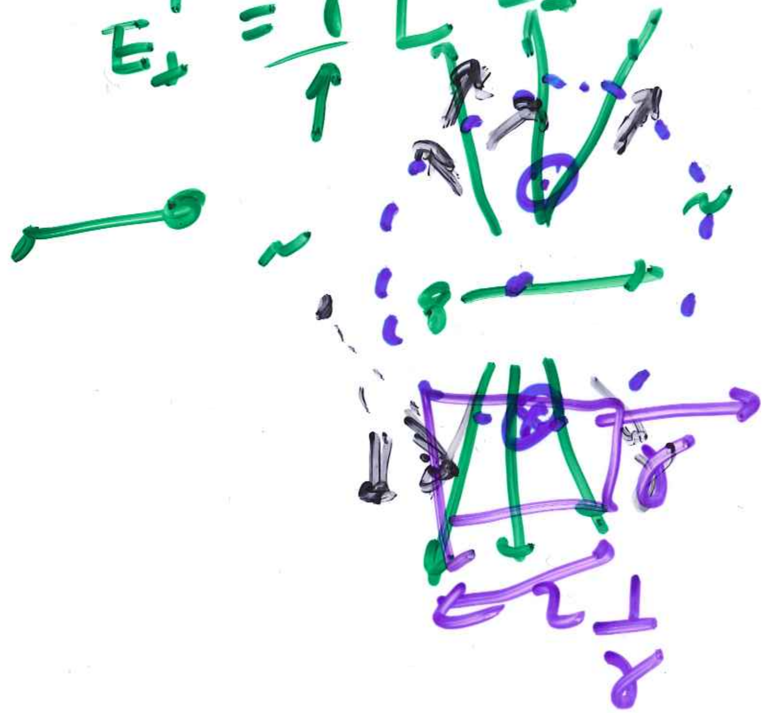
$$F_{\mu\nu} U_\nu$$

dual $F = \tilde{F}$

$$\partial_\nu \tilde{F}_{\mu\nu} = 0$$

$$F'_{\mu\nu} = R\, F_{\mu\nu}\, R^{T} \ldots \qquad \underline{HW}$$

$$E'_{\perp} = \gamma \left[E_{\perp} + \vec{v} \times \vec{B} \right] \ldots$$



$E' =$ Columb

$B' = 0$

Thus 10:50

Help? Tues 7PM