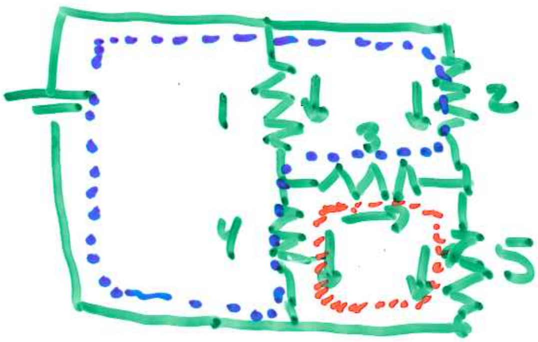
In circuits with single frequency sinusoidal drives we can treat L & C as if R but with complex "resistance" = impedance $i\omega L$ & $\frac{1}{i\omega C}$

IF drive is not sinusoidal use superposition & Fourier

Kirchhoff : ① $\sum V = 0$ ② $\sum I = 0$



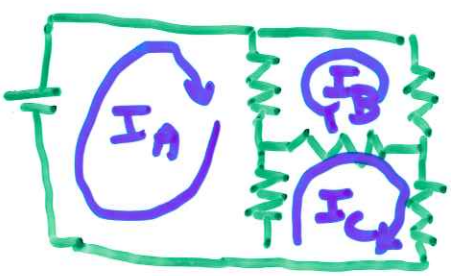
$V - I_2R_2 + I_3R_3 - I_4R_4 = 0$

$-I_5R_5 + I_4R_4 - I_3R_3 = 0$

$I_1 - I_3 - I_4 = 0$

$I_2 + I_3 - I_5 = 0$

Mesh: imagine continuous loop currents that superposition to form actual current

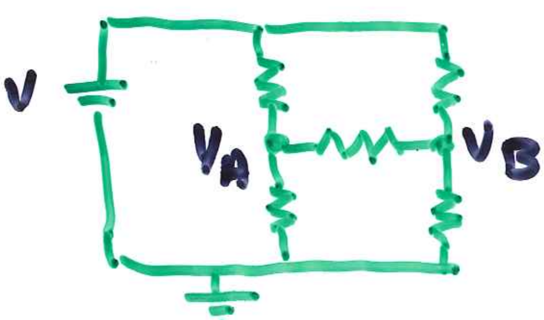


"I_1" = $I_A - I_B$

"I_3" = $I_C - I_B$

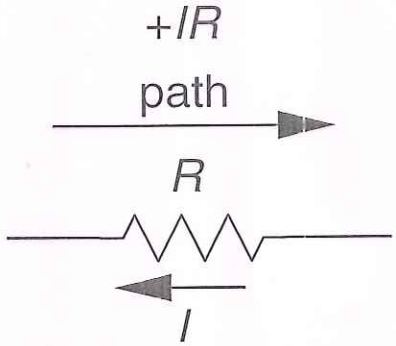
"I_2" = I_B

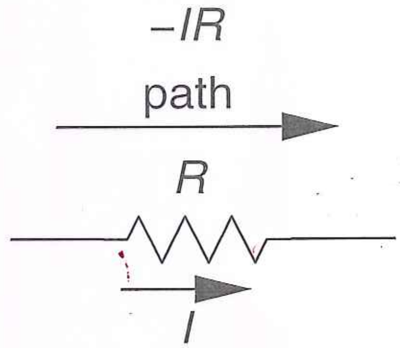
Node: assign a ground ($V=0$) & unknown voltages to every junction; write $\sum I = 0$

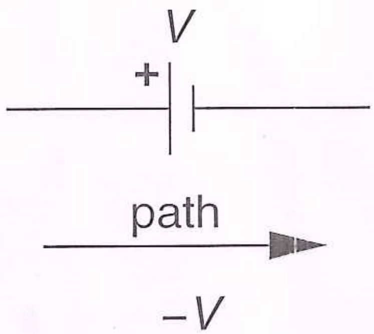


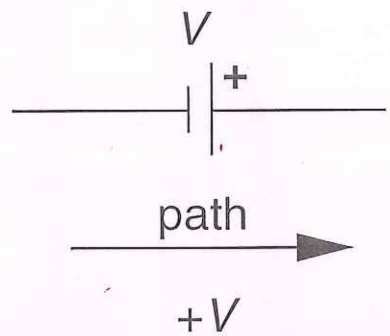
$$\frac{(V - V_A)}{R_1} + \frac{(V_B - V_A)}{R_3} + \frac{(0 - V_A)}{R_4} = 0$$

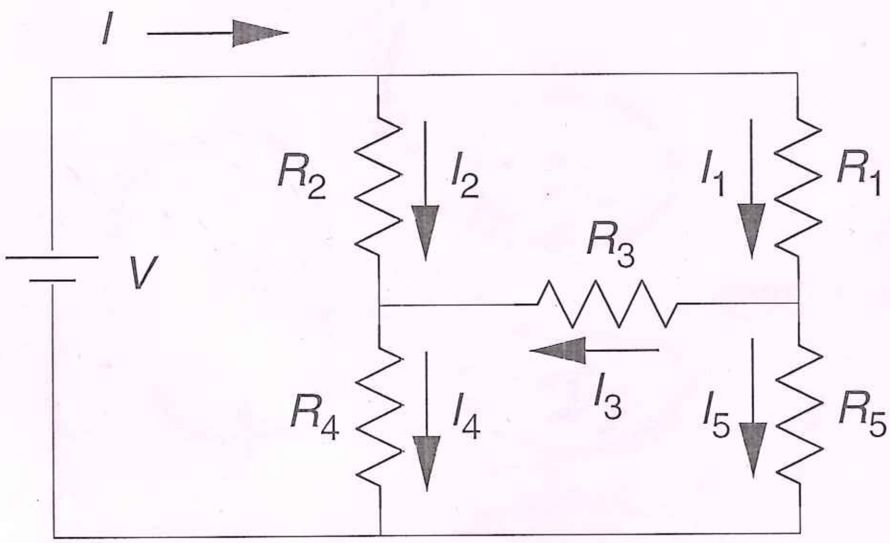
$$\frac{(V - V_B)}{R_2} + \frac{(V_A - V_B)}{R_3} + \frac{(0 - V_B)}{R_5} = 0$$

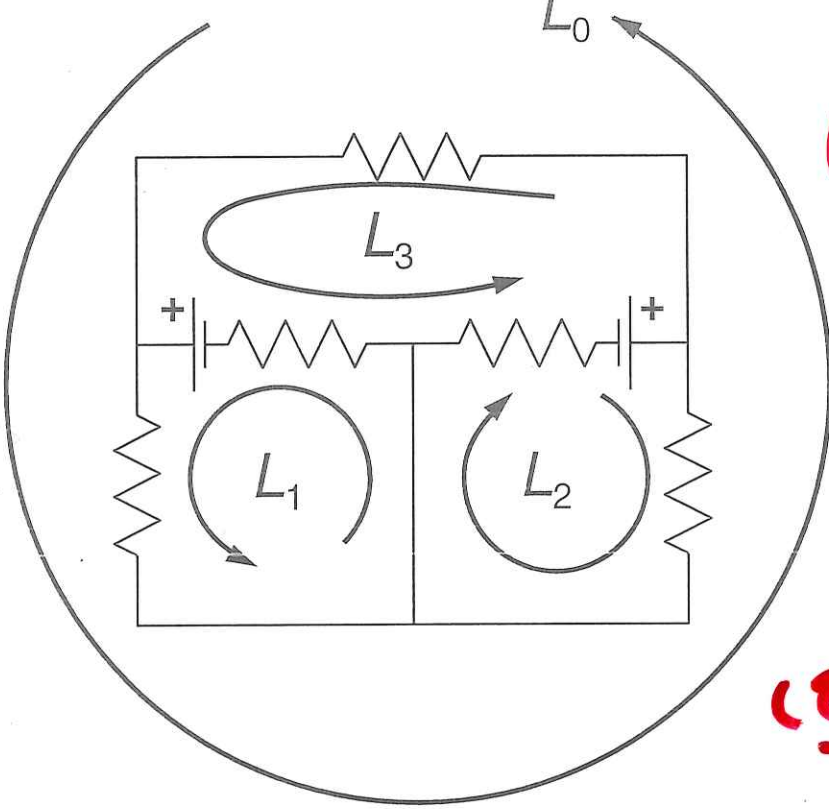

+IR
path
R
I


−IR
path
R
I

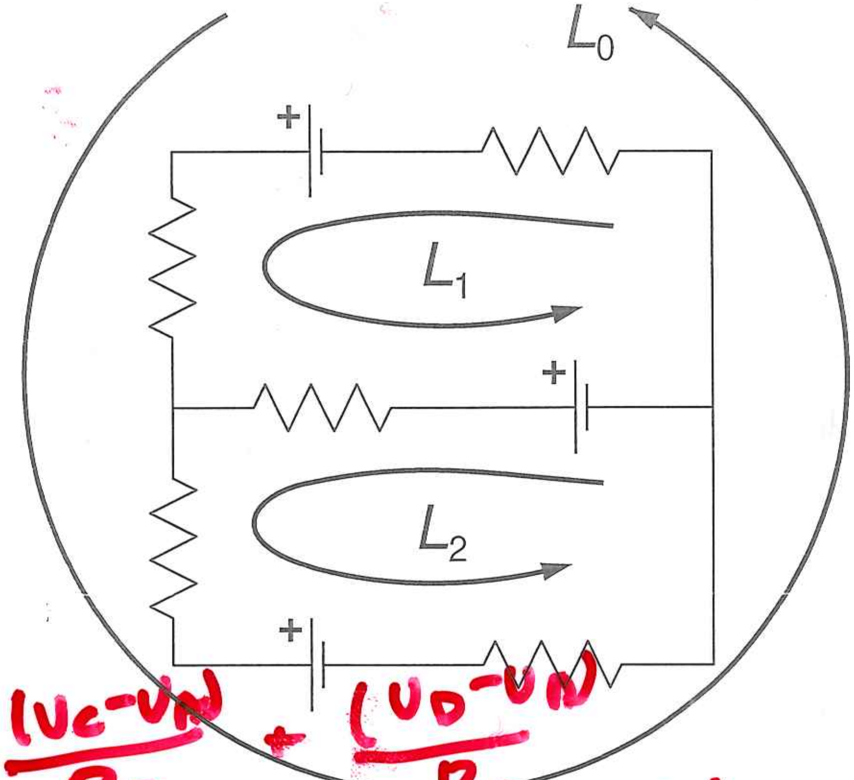

V
+
path
−V


V
+
path
+V


I
V
R2
I2
I1
R1
R3
I3
R4
I4
I5
R5

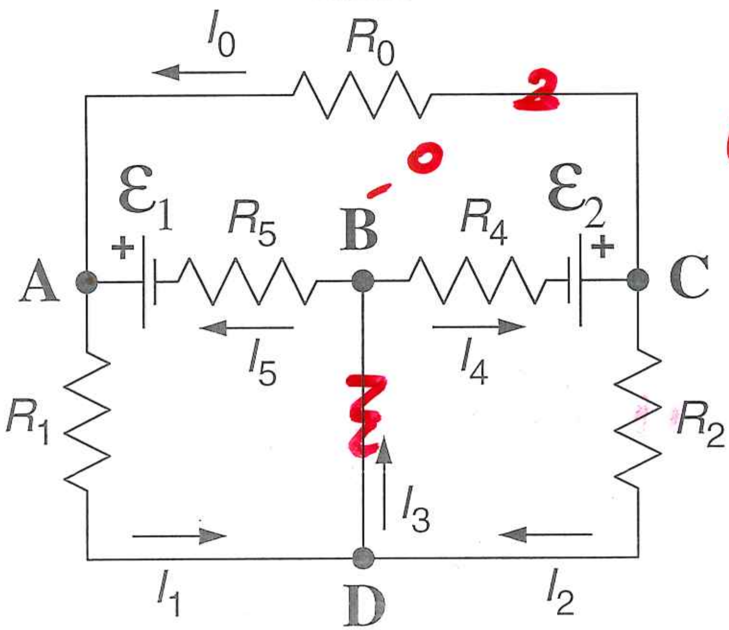


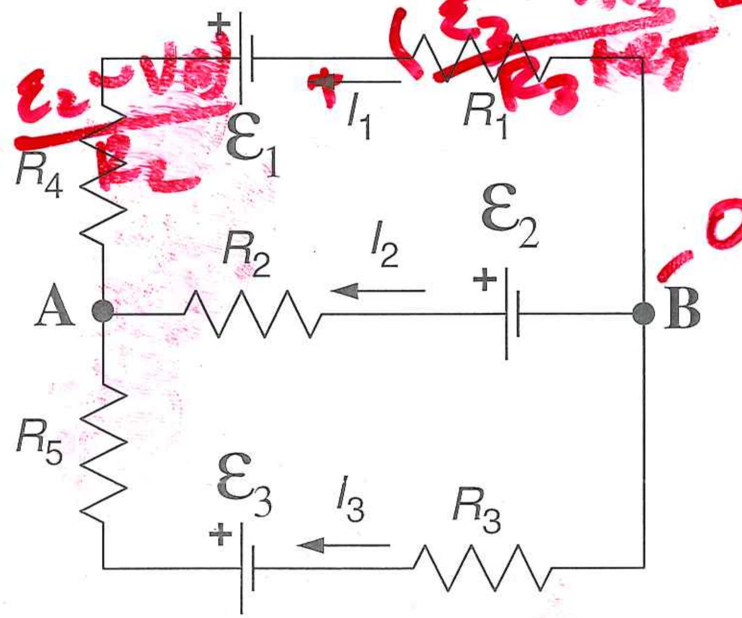
FET



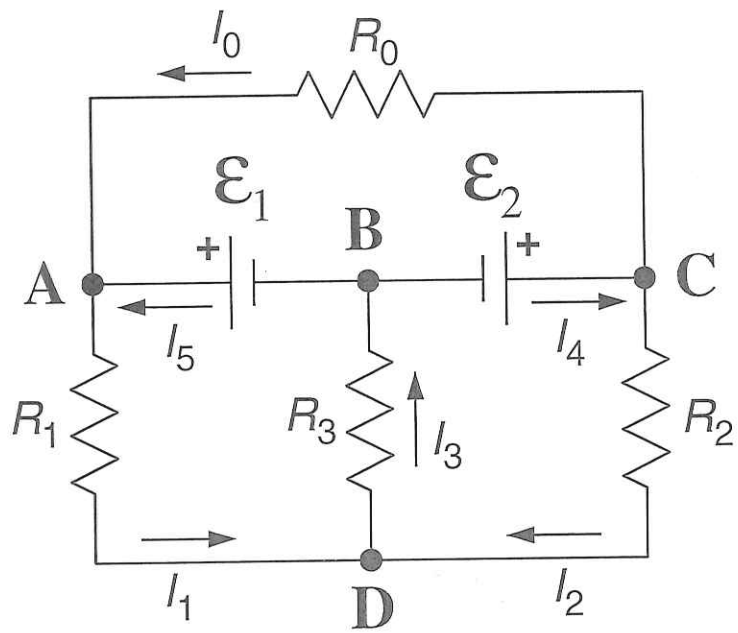
$$\frac{(\mathcal{E}_1 - V_A)}{R_5} + \frac{(V_C - V_A)}{R_0} + \frac{(V_D - V_A)}{R_1}$$

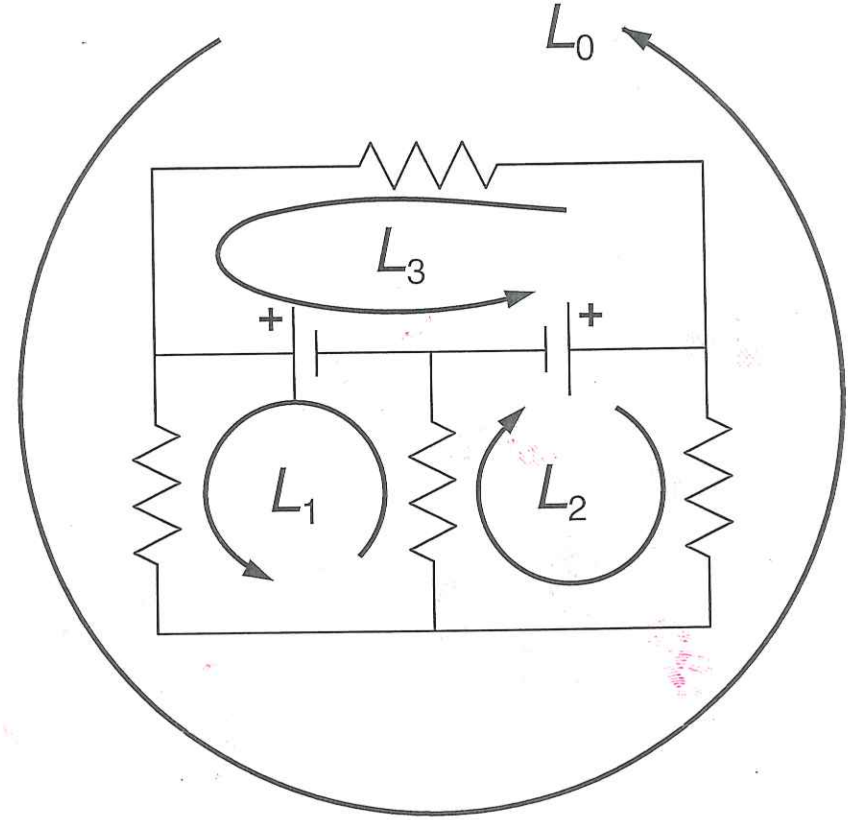
$$\frac{(\mathcal{E}_1 - V_A)}{R_1 + R_4} + \frac{(\mathcal{E}_2 - V_A)}{R_2} + \frac{(\mathcal{E}_3 - V_A)}{R_3 + R_5} = 0$$





$$\frac{(\varepsilon_1 - V_D)}{R_1} + \frac{(\varepsilon_2 - V_D)}{R_2} + \frac{(0 - V_D)}{R_3} = 0$$





Power $\quad P = VI$

$$\bar{P} = \overline{V I_R}$$

$V \leftarrow V_0 \cos(\omega t + \phi)$ $\qquad$ $I_R \leftarrow I_0 \cos(\omega t)$

$$\cos(\omega t + \phi) = \cos(\omega t)\cos\phi - \sin(\omega t)\sin\phi$$

$$\bar{P} = V_0 I_0 \left(\underbrace{\overline{\cos^2(\omega t)}}_{\frac{1}{2}} \cos\phi - \underbrace{\overline{\sin(\omega t)\cos(\omega t)}}_{0} \sin\phi \right)$$

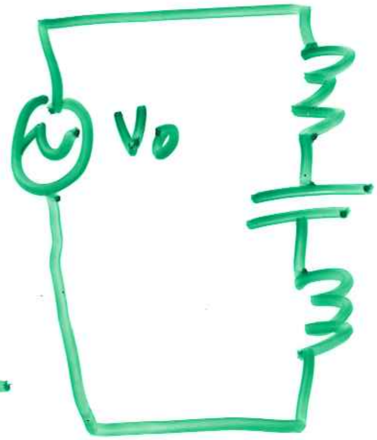
$$= \frac{1}{2} V_0 I_0 \cos\phi \qquad V_{rms} = \frac{V_0}{\sqrt{2}}$$

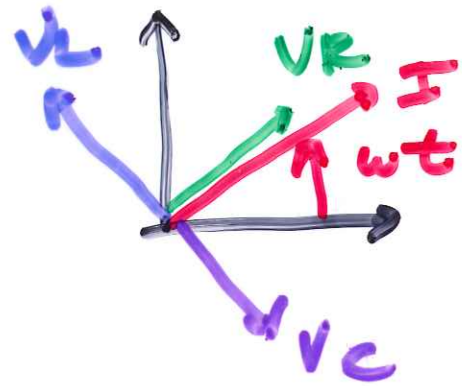
$$= V_{rms} I_{rms} \underbrace{\cos\phi}_{\text{power factor}}$$

$\phi = -90^\circ$ $\qquad$ $120\ I_{rms}$

$\phi = 90$

Resonance





$$Z = i\omega L + \frac{1}{i\omega C} + R$$

$$= i\left(\omega L - \frac{1}{\omega C}\right) + R$$

$$V = V_0 e^{i\omega t}$$

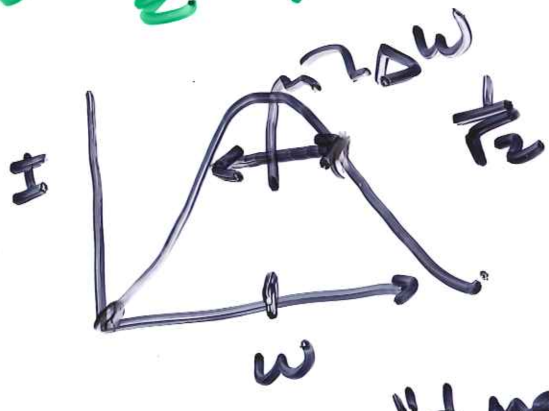
$$I = I_0 e^{i\omega t}$$

$$e^{i\omega t}$$

$$|Z| e^{i\phi}$$

$$V = IZ \longrightarrow V_0 = Z I_0$$

$$I_0 = \frac{V_0}{Z} \; ; \qquad |I_0| = \frac{|V_0|}{|Z|} = \frac{|V_0|}{\sqrt{R^2 + \underbrace{\left(\omega L - \frac{1}{\omega C}\right)^2}_{R}}}$$



full width "$\frac{1}{2}$ max"

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$f(x_0 + \Delta x) = f(x_0) + f'(x_0)\Delta x + \dots$$

$$\left(\omega L - \frac{1}{\omega C}\right)\Big|_{\omega_0 + \Delta\omega} = R$$

$$0 + \left(L + \frac{1}{\omega^2 C}\right)\Big|_{\omega_0} \Delta\omega = R$$

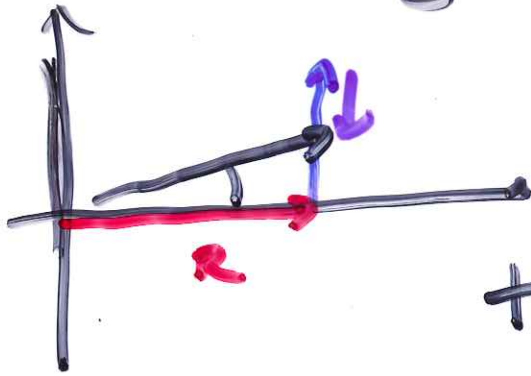
$2L$

$$\Delta\omega = \frac{R}{2L}$$

$$2\Delta\omega = \frac{R}{L}$$

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0}{R/L} \Rightarrow \frac{\omega_0 L}{R} = \frac{1}{R\omega_0 C}$$

$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

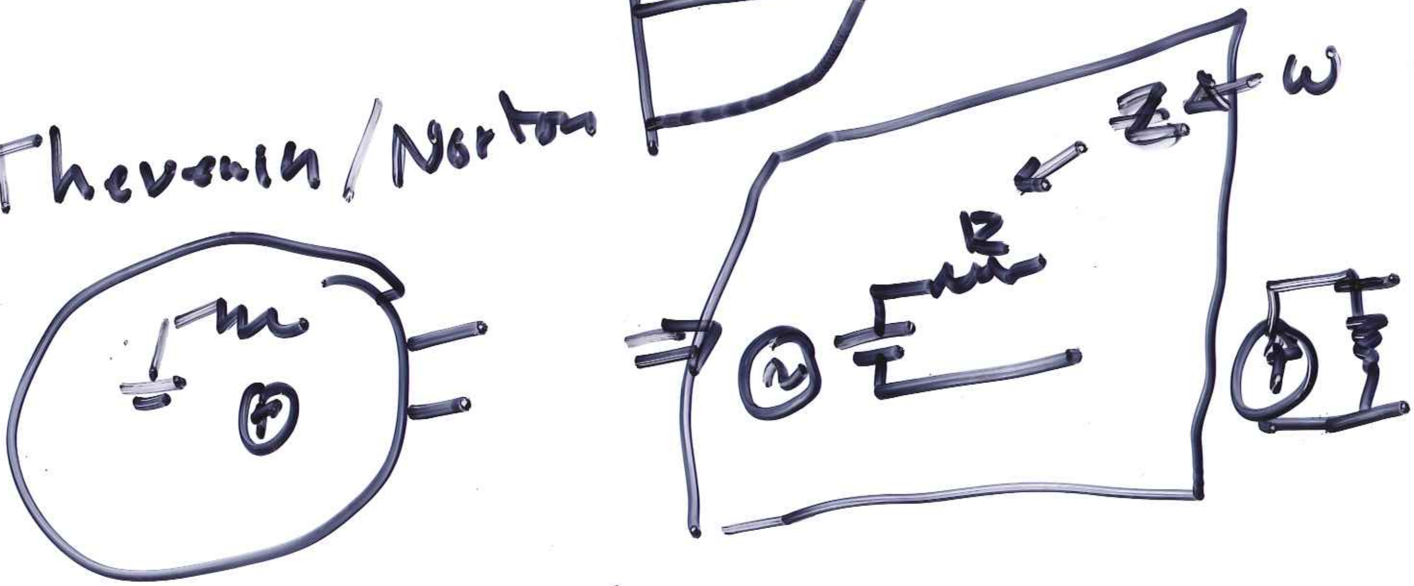


$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$\omega \to 0 \quad \phi = -90^\circ \leftarrow \phi = 0$

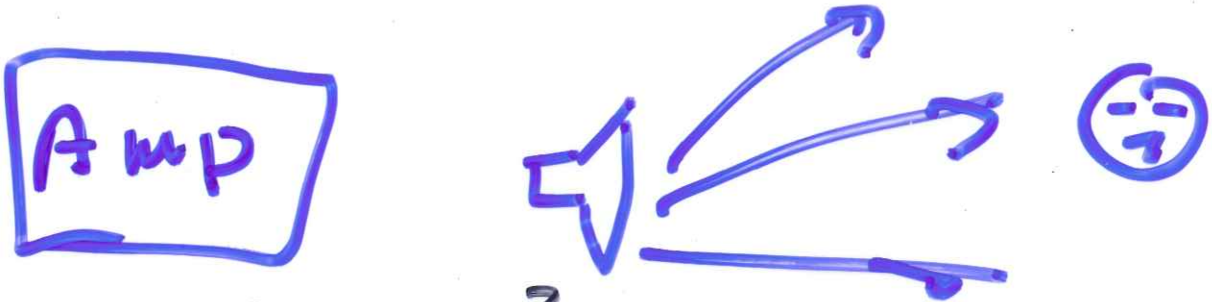
$\omega \to \infty \quad \phi = 90^\circ$

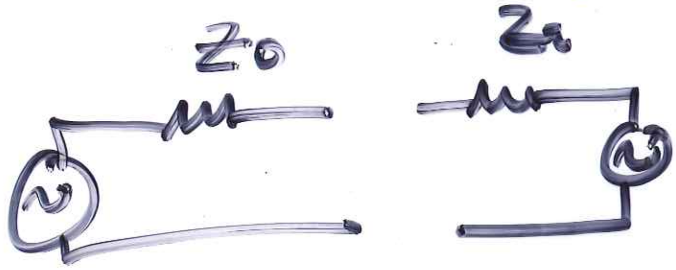
Thevenin / Norton

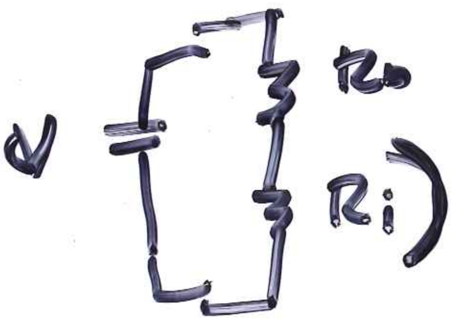


output impedance

input impedance







$$I = \frac{V}{R_o + R_i}$$

$$\frac{V^2}{(R_o+R_i)^2} R_i \rightarrow x$$

$$\frac{V^2}{R_o} \frac{(R_i/R_o)}{\left(1+\left(\frac{R_i}{R_o}\right)\right)^2}$$

$R_i \to 0$

$R_i \to \infty$

$$\frac{x}{(1+x)^2} \quad ; \quad \frac{1}{(1+x)^2} - 2\frac{x}{(1+x)^3}$$

$$\frac{1+x-2x}{(1+x)^3} = \frac{1-x}{(1+x)^3} = 0$$

$$x = 1 \rightarrow R_i = R_o$$

$$Z_i^* = Z_o$$

$$\overline{Re(v)\, Re(I)} = \frac{1}{2} Re\, V^* I$$

Im

$f = f_0 e^{i\omega t}$

$g = g_0 e^{i\omega t}$

$Re(f)\, Re(g)$

$f_0 = a + ib \qquad g_0 = c + id$

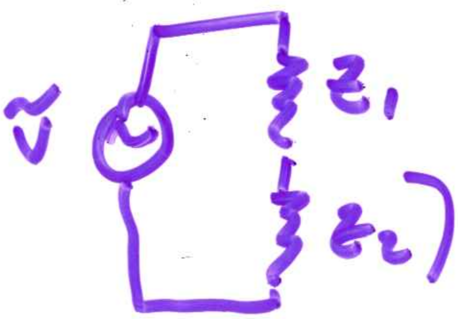
$$(a\cos(\omega t) - b\sin(\omega t))(c\cos(\omega t) - d\sin(\omega t))$$

$$= \frac{1}{2}(ac + bd)$$

$f^* = (a - ib)\, e^{-i\omega t}$

$g = (c + id)\, e^{i\omega t}$

$ac + bd$

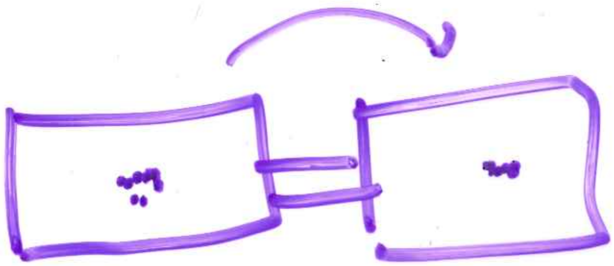


$$I = \frac{\tilde{V}}{Z_1 + Z_2}$$

$$V = \frac{Z_2}{Z_1 + Z_2}\tilde{V}$$

$$\frac{1}{2}\frac{1}{(Z_1+Z_2)^*}\frac{Z_2}{Z_1+Z_2} \qquad Im(Z_1 + Z_2) = 0$$

$$\frac{R_2}{(R_1+R_2)^2}$$



$$Z_i \gg Z_o$$