$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times (\vec{r-r'})}{|\vec{r-r'}|^3}$$

$\vec{J}\, dV; \; \vec{K}\, da$

$\nabla \cdot B = 0$ no monopoles

$\nabla \times A = B$

Gauge invariance

$SU(3) \times SU(2) \times U(1)$

Gravity

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{J\, dV}{|r-r'|}$$

$\nabla \times B = \mu_0 J \rightarrow \vec{B} = -\nabla \frac{\mu_0 I}{4\pi} \Omega \leftarrow$

$$\vec{M} = \frac{\sum \vec{m}_i}{\text{Volume}} \quad \ldots \quad J_b = \nabla \times M, \quad K_b = M \times n$$

$M = \chi H \leftarrow$ approx $\qquad M \to P$

$$B = \mu_0 (H + M) = \underbrace{\mu_0 \overbrace{(1+\chi)}^{K_m}}_{\mu} H$$

$\nabla \times E = -\partial_t B \rightarrow -\frac{d\Phi_m}{dt} = \mathcal{E} = \oint E \cdot dl$

$\nabla \times B = \mu_0 J + \partial_t E$

rhr

Lenz

$\Phi_i = \int \vec{B} \cdot d\vec{A}$

$= \sum M_{ij} I_j \qquad M_{ii} = L_i$

Mutual & Self inductance: at any point in space the magnetic field is the superposition of source currents — the source currents are linearly related to the resulting B via Biot-Savart. Therefore if we have a system of N current loops (each with current I_i) the flux thru the ith loop must be linear in the I_i

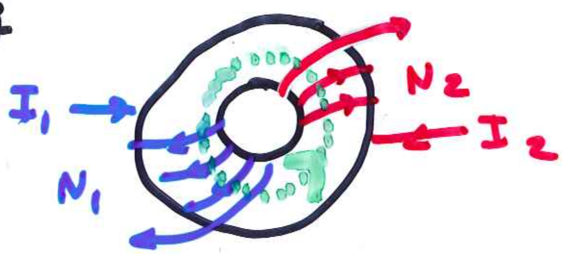
$$\phi_i = \sum M_{ij} I_j$$

$L_i \equiv M_{ii}$ <u>self</u>
otherwise mutual

$$M_{21} = \int_{S_2} \vec{B}(r_2) \cdot dA_2 \qquad \frac{\mu_0 I_1}{4\pi} \oint_{L_1} \frac{d\vec{r}_1 \times (\vec{r_2 - r_1})}{|\vec{r_2 - r_1}|^3}$$

$$= \frac{\mu_0}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\vec{r}_1 \cdot d\vec{r}_2}{|\vec{r_2 - r_1}|} \Rightarrow M_{ij} = M_{ji}$$

Eg



Amp Loop

$$\underbrace{\oint H \cdot dl}_{H\,2\pi r} = (N_1 I_1 + N_2 I_2)$$

ignor
$2\pi r = l = \text{const}$

$$\phi = BA = \frac{\mu}{l} A (N_1 I_1 + N_2 I_2)$$

one loop

$$\phi_1 = N_1 \phi = \frac{\mu A}{l} N_1 (N_1 I_1 + N_2 I_2)$$

$$\phi_2 = N_2 \phi = \frac{\mu A}{l} N_2 (N_1 I_1 + N_2 I_2)$$

so: $M_{12} = M_{21} = \frac{\mu A}{l} N_1 N_2$

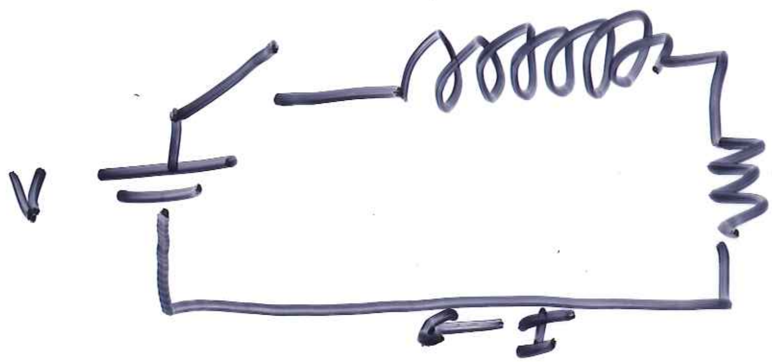
$L_1 = \frac{\mu A}{l} N_1^2$

$L_2 = \frac{\mu A}{l} N_2^2$

transformer: $\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$ if $I_2 = 0$
motional emf

Energy ... $\underbrace{\frac{1}{2}\rho\phi}_{\text{charge position}}$ ... $\underbrace{\frac{1}{2}\epsilon\cdot D}_{\text{Fields as Energy}}$

$\frac{1}{2}J\cdot A$ $\qquad$ $\frac{1}{2}B\cdot H$



$$\mathcal{E} = -\frac{d\phi}{dt}$$

$$V = L\frac{dI}{dt}$$

$$V - L\frac{dI}{dt} = IR$$

$$I = \frac{V}{R}$$

$$V = L\frac{dI}{dt} + IR$$

$$I = I_h + I_p$$

$$0 = L\frac{dI}{dt} + IR$$

$$I = I_0 e^{-t/\tau} + \frac{V}{R}$$

$$= \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}$$

$$-\frac{R}{L}I = \frac{dI}{dt}$$

$$-\frac{R}{L}dt = \frac{dI}{I}$$

$$-\frac{R}{L}t = \ln I$$

$$I = I_0 e^{-\frac{R}{L}t}$$

$$I = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

$$\text{Power} = VI = \frac{V^2}{R} - \frac{V^2}{R} e^{-t/\tau}$$

$$\left(V - \frac{d\phi}{dt} = IR\right) I$$

$$VI = I^2 R + \underbrace{I \frac{d\phi}{dt}}_{\text{Magnetic Energy}}$$

$$du = I \, d\phi$$

mag en

flux

$$d\phi_i = \sum M_{ij} \, dI_j$$

$$\sum I_i \, d\phi_i = \sum M_{ij} I_i \, dI_j$$

$$\parallel$$

$$d\left(\begin{matrix}\text{total} \\ \text{Mag} \\ \text{Energy}\end{matrix}\right)$$

$$I_i(t) = \alpha I_i$$

$$0 \to 1$$

$$\int_0^1 \sum M_{ij} \, \alpha I_i \, d\alpha \, I_j$$

$$u = \frac{1}{2}\sum M_{ij} I_i I_j$$

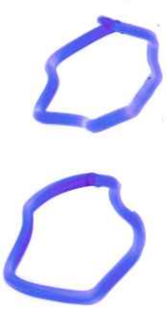
$$\int \alpha\, d\alpha = \frac{1}{2}\alpha^2\Big|_0^1 = \frac{1}{2}$$

$$= \frac{1}{2}\sum_i I_i \Phi_i$$

$$\Phi_i = \int_{S_i} B \cdot dA$$

$$B = \nabla \times \overline{A}$$

$$\oint_{C_i} \overline{A} \cdot d\overline{l}$$



$$= \frac{1}{2}\sum_i \oint_{C_i} \overline{A} \cdot \underbrace{I_i\, dl}_{J\, dV}$$

$$= \frac{1}{2}\int \overline{A} \cdot J\, dV = \frac{1}{2}\int B \cdot H\, dV$$

$$J = \nabla \times H$$

$$A \cdot \nabla \times H = \nabla \cdot (A \times H) + \overbrace{(\nabla \times A)}^{B} \cdot H$$

$$\nabla \cdot (F \times G) = \underline{(\nabla \times F)} \cdot G - (\nabla \times G) \cdot F$$

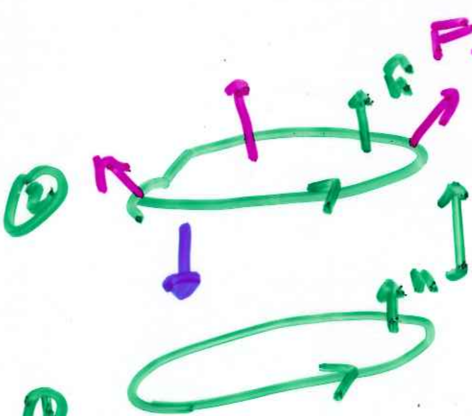
$$\int \nabla\cdot(A\times H)\,dV = \int A\times H\cdot dS$$

$$A \sim 1/r^2 \qquad H \sim 1/r^3$$

$$= \int \frac{1}{r^5}\,dS \propto \frac{1}{r^3}$$

$F = -\nabla U$ ← Φ const

$= +\nabla \hat{U}$ ← currents const

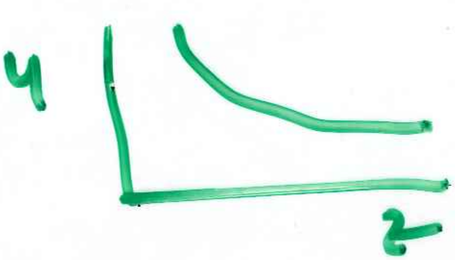


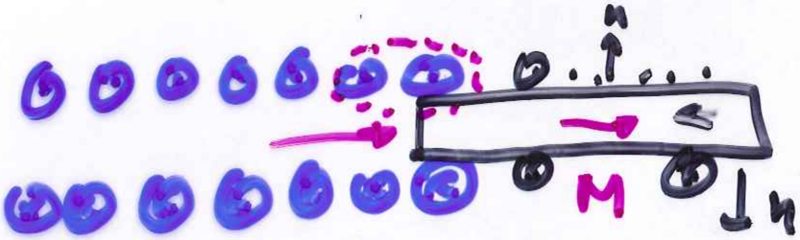
← $\Phi_2 = \underset{(+)}{M_{21}} I_1$

$$U = \frac{1}{2}\sum M_{ij} I_i I_j$$

$$= \frac{1}{2} M_{11} I_1^2 + \frac{1}{2} M_{12} I_1 I_2 + \frac{1}{2} M_{21} I_2 I_1 + \frac{1}{2} M_{22} I_2^2$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2$$





$K = M \times \hat{n}$

$B = \mu_0 \frac{N}{\ell} I$ $\qquad$ $H = \frac{N}{\ell} I$

$\phi_1 = \mu_0 \frac{N}{\ell} A I$ $\qquad$ $H \Delta x = \frac{N}{\ell} \Delta x \, I$

$L = \mu_0 \frac{N^2}{\ell} A$ $\qquad$ $B = \mu \frac{N}{\ell} I$

$L \uparrow$ $\qquad$ $U = \frac{1}{2} L I^2$

$U \uparrow$

$dU = F \cdot d\ell = -dU_m + dW_b$

$\sum I \, d\phi$

$U = \frac{1}{2} \sum I_i \phi_i$

$dU = \frac{1}{2} \sum I_i \, d\phi$

$= +\frac{1}{2} \sum I \, d\phi$

$= U + dU_m$

$F = +\frac{d}{dz} U$

$\tau = +\frac{d}{d\theta} U$

→ p^+ $\qquad\qquad$ $\frac{3}{5}\frac{e^2}{4\pi\epsilon_0 R}$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\frac{1}{2} B \cdot H = \mu_0 \left(\frac{I}{2\pi r}\right)^2 \frac{1}{2}$$

$$\int_{R}^{\infty} \frac{1}{r^2}\, 2\pi r l \, dr \qquad \ln(\infty) - \ln(R)$$

$$B\, 2\pi r = \mu_0 I \frac{r^2}{R^2}$$

$$B \propto r$$

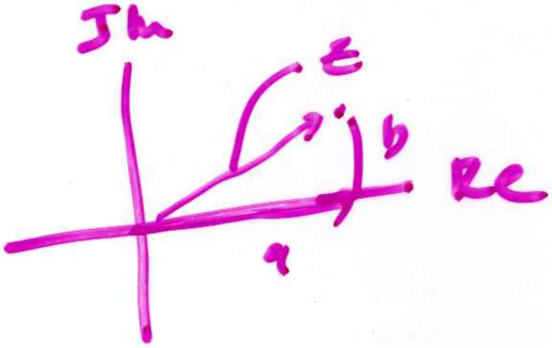
$$\int r^2\, 2\pi r l \, dr$$

Circuits; Kirchhoff; Impedance; Complex #5

$i = \sqrt{-1}$ $\qquad z = a + bi = (a, b)$

$\mathbb{C}$ $\qquad\qquad \mathbb{R}$

$$z_1 \cdot z_2 = (a+bi)(c+di)$$
$$= ac - bd + i(ad + bc)$$



$z = 2 + 3i$

$|z| = \sqrt{4+9}$

$$|z| \cos\theta + |z| \sin\theta \, i$$

$$|z| \big(\underbrace{\cos\theta + i \sin\theta}_{e^{i\theta}} \big)$$

$\sqrt{a^2 + b^2}$