

with dielectrics your knee-jerk reaction:  $\epsilon_0 \rightarrow \epsilon$

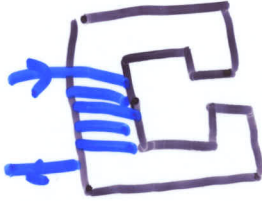
(but don't go too far: Gauss is either

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{total}}{\epsilon_0} \quad \text{or} \quad \oint \vec{D} \cdot d\vec{A} = Q_F \quad \text{NOT} \quad \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon}$$

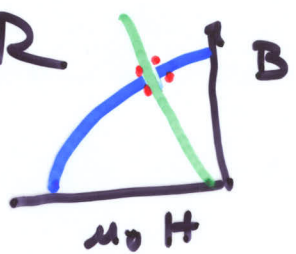
Similarly  $\mu_0 \rightarrow \mu$  is usual

Note: for dielectrics we often had only  $\rho_B$   
 as:  $\rho_B = -\nabla \cdot \mathbf{P} = -\nabla \cdot \epsilon_0 \chi \mathbf{E} = -\frac{\chi}{\kappa} \nabla \cdot \mathbf{D} = -\frac{\chi}{\kappa} \rho_F$   
 and  $\rho_F$  was often zero inside dielectric

Similarly:  $\mathbf{J}_B = \nabla \times \mathbf{M} = \chi \nabla \times \mathbf{H} = \chi \mathbf{J}_F$   
 and  $\mathbf{J}_F$  is often zero inside magnetic material

Magnetic Circuits   $\leftarrow \text{gap; } \delta$   
 $\oint \vec{H} \cdot d\vec{\ell} = NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{\ell} = \phi \int \frac{d\ell}{\mu A} = \phi R_{tot}$   
 Flux  $\approx BA$   
 "I"  $\uparrow$  reluctance "R"

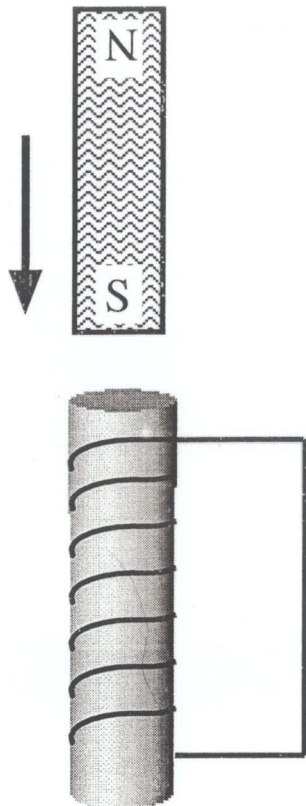
$$R_{tot} = \sum \frac{\ell_i}{\mu_i A_i} \quad ; \text{ if dominated by gap } \rightarrow B = \mu_0 \frac{NI}{\delta}$$

"flux return"  $\rightarrow$  gap  $= \infty$  solenoid  $\rightarrow$   
 PM  $H_{PM} \ell_{PM} = -\phi R = -B_{PM} A_{PM} R$   
 $B_{PM} = \frac{-\ell_{PM}}{A_{PM} R} H_{PM}$  "Keeper"  


Note: Egs not exact due to uncertainty  $\ell, \mu, A$

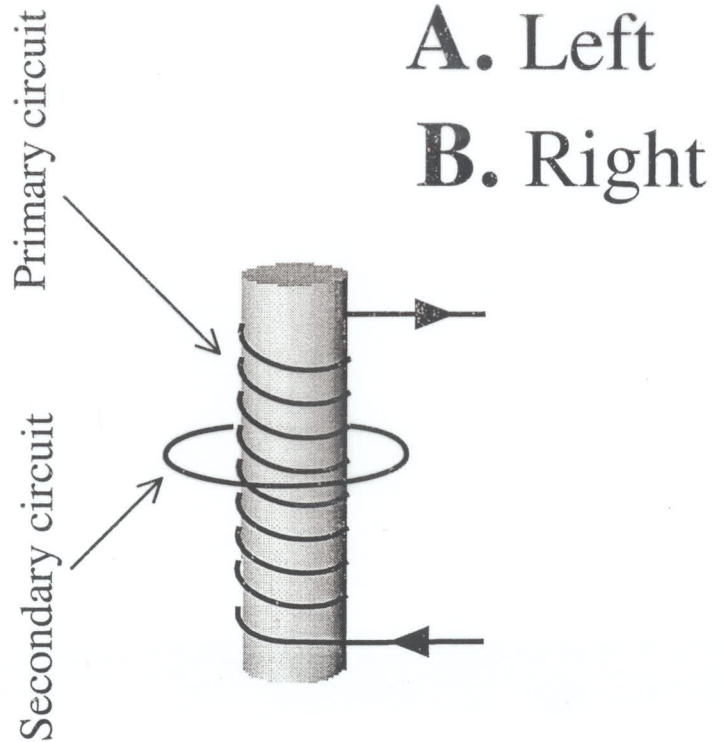
Induction:  $-\frac{d}{dt} \phi = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = \mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\int \nabla \times \vec{E} \cdot d\vec{A}$

$\therefore -\partial_t \mathbf{B} = \nabla \times \mathbf{E}$  signs!  
 $\mathbf{E} = -\nabla \phi - \partial_t \mathbf{A}$



The south pole of a bar magnet is approaching the solenoid

**A. Up**    **B. Down**



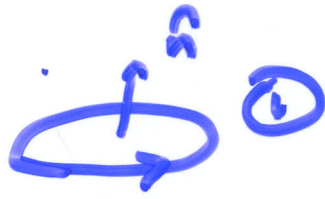
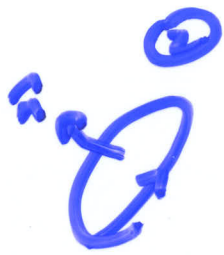
The current shown flowing through the primary circuit is increasing

**A. Left**  
**B. Right**



**A. Clockwise**  
**B. CCW**

A long straight wire and a square loop of wire sit in the plane of this sheet of paper. A circular loop of wire is centered on the long wire, but sits in a plane perpendicular to this sheet of paper. For several minutes there has been no current flowing in any wire. A battery (not shown) is connected and a current starts to flow to the right through the long straight wire.



$$\vec{B} = \sum \vec{B}_i = \sum \underline{\underline{I_i}}$$

$$\phi = \int \vec{B} \cdot d\vec{A} \quad \text{inductances}$$

$$\phi_i = \sum M_{ij} I_j$$

$$M_{ii} = L_i \quad \begin{array}{l} \text{self inductance} \\ \text{mutual inductance} \end{array}$$

$$\begin{aligned} B(\vec{r}_2) &= \frac{\mu_0 I_1}{4\pi} \int d\vec{r}_1 \times \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3} \\ &= \nabla_2 \times \vec{A} \quad \frac{\mu_0 I}{4\pi} \int \frac{d\vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \end{aligned}$$

$$\phi_2 = \int_{S_2} B(\vec{r}_2) \cdot d\vec{A}_2$$

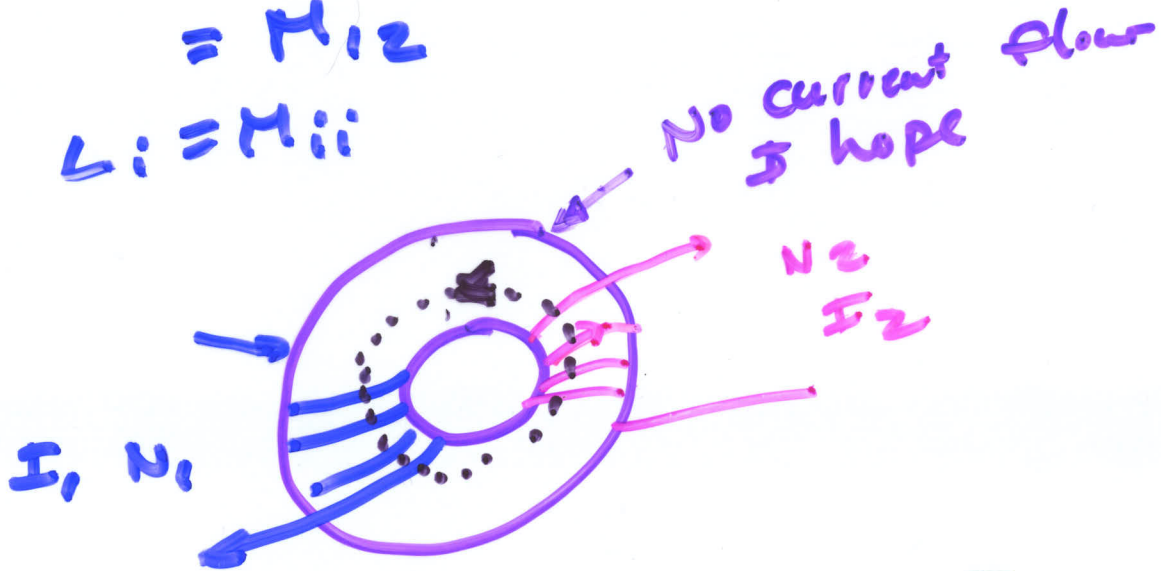
$$M_{21} = \frac{\phi_2}{I_1} = \frac{\mu_0}{4\pi} \int B(\vec{r}_2) \cdot d\vec{A}_2$$

$$= \frac{\mu_0}{4\pi} \int \nabla_2 \times \int \frac{d\vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \cdot d\vec{A}_2$$

$$= \frac{\mu_0}{4\pi} \oint_2 \oint_1 \frac{d\vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \cdot d\vec{r}_2$$

$$= M_{12}$$

$$L_i = M_{ii}$$



$$\oint \vec{H} \cdot d\vec{l} = N_1 I_1 + N_2 I_2$$

" " "average r"

$$\rightarrow H \underbrace{2\pi r} \rightarrow \frac{B}{\mu}$$

$$\frac{B}{\mu}$$

$$B = \frac{\mu}{l} (N_1 I_1 + N_2 I_2)$$

flux 1 loop =  $\underline{BA} = \int B dA$

$$\Phi_1 = N_1 BA = \frac{\mu A}{l} (N_1^2 I_1 + N_1 N_2 I_2)$$

$$L_1 = \frac{\mu A}{l} N_1^2$$

$$M_{12} = \frac{\mu A}{l} N_1 N_2$$

$$\Phi_2 \Rightarrow L_2 = \frac{\mu A}{l} N_2^2 \quad M_{21} =$$

$$M_{21} = k \sqrt{L_1 L_2}$$

$\uparrow$   
 $1 \leftarrow 0 \rightarrow 2$

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -L_2 \frac{dI_2}{dt} - M_{21} \frac{dI_1}{dt}$$

$\uparrow$   
 $\text{emf} = V$

$$\mathcal{E}_1 = -L_1 \frac{dI_1}{dt} - M_{12} \frac{dI_2}{dt}$$

$\rightarrow 0$   $\Phi_2 = 0$  ✓

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{-M_{12} \frac{dI_1}{dt}}{-L_1 \frac{dI_1}{dt}} = \frac{N_1 N_2}{N_1^2} = \frac{N_2}{N_1}$$

$\uparrow$   
 $\frac{N_2}{N_1}$

Transformer  
 non-linear.

$\mathcal{E}_2 \gg \mathcal{E}_1$   
 $N_2 \gg N_1$

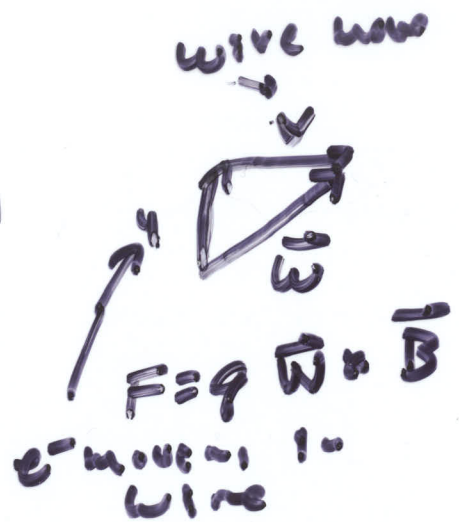
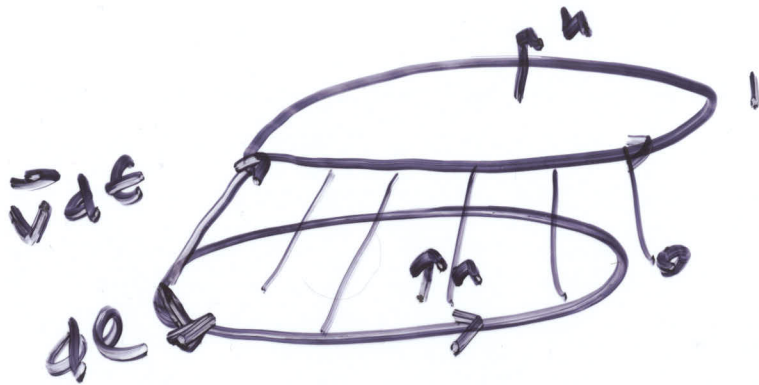
$\frac{1}{2} B \cdot D$   
 $\uparrow$   
 $\frac{1}{2} B \cdot \sigma$

Ferrites

Eddy currents



Method EHF.



$\Delta \phi = \phi_1 - \phi_0 = \int_{\text{ribbon}} \vec{v} \times d\vec{\ell} \cdot d\vec{A}$

$$\Delta \phi = \int_B \vec{v} \times d\vec{\ell} \cdot d\vec{A}$$

$\vec{v} = \vec{\omega} \times \vec{r}$   
?

$\vec{v} = \vec{\omega} \times \vec{r}$   
 $u \times d\vec{\ell} = 0$

$$= \int_B \vec{\omega} \times d\vec{\ell} \cdot d\vec{A}$$

$$= \int (\vec{B} \times \vec{\omega}) \cdot d\vec{\ell}$$

$$= \int (\vec{\omega} \times \vec{B}) \cdot d\vec{\ell}$$

$$\frac{d\phi}{dt} = \int \vec{f} \cdot d\vec{\ell} = \sum \text{force/charge}$$