Continuity Equation (conservation of charge)

$$\partial_t \rho + \vec{\nabla}\cdot\vec{J} = 0$$

Note: in most conductors $\partial_t \rho \xrightarrow{\text{quickly}} 0$

$\Rightarrow \nabla\cdot J = 0 \underset{J = gE}{\Longrightarrow} \nabla\cdot E = 0 \Rightarrow \nabla^2\phi = 0$ Laplace

BC: $\phi^{in} = \phi^{out}$

usual Laplace methods for flowing currents

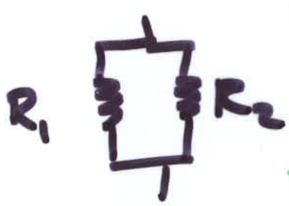
$\underbrace{g_{in}\partial_r\phi^{in}}_{J_r^{in}} = \underbrace{g_{out}\partial_r\phi^{out}}_{J_r^{out}}$

Resistors $V = IR \leftarrow \frac{\eta l}{A}$

$P = IV = I^2 R = \frac{V^2}{R}$

Parallel:

Is add



V same

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Series

I same

Vs add

$R = R_1 + R_2$

$\vec{B} = \frac{\mu_0}{4\pi}\oint I\,d\vec{l}\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$

$\vec{F} = \oint I\,d\vec{l}\times\vec{B}$

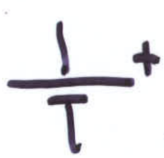
$I\,d\vec{l},\ \vec{K}\,dA,\ \vec{J}\,dV$

Kirchhoff

① Junction ~~Loop~~: Current in = Current out

② Loop: $\sum \Delta V = 0$

+V



-V

I

$+IR$

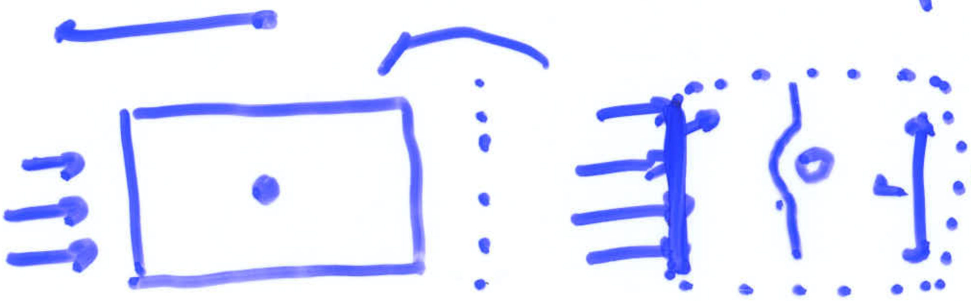
$-IR$

rref

$J = q N v_D = \sigma E$

$N = \frac{\#\ \text{part}}{\text{volume}}$

$\phi \xrightarrow{d} E \to J$



avg V

$\frac{1}{L}\int V\,dy$

$V \leftarrow \phi$

$I = \int J_x\,dy$

$R = \frac{V}{I}$

$$\phi = \sum_{n \geq 1} \left(A_n r^n + \frac{C_n}{r^n} \right) \cos(n\theta) - E_0 x$$

$\phi \leftarrow \ln r$

$x = r\cos\theta$

$\vec{r}\,' = \langle 0, 0, z')$

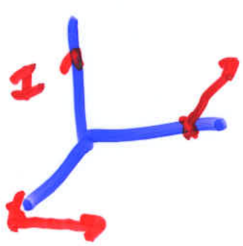
$\vec{r} = (x, 0, 0)$ wolog

$d\vec{\ell}\,' = dz'\,\hat{k}$

$|\vec{r} - \vec{r}\,'| = |(x, 0, -z')| = \sqrt{x^2 + z'^2}$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\hat{k}\, dz' \times (x, 0, -z')}{\sqrt{x^2 + z'^2}^{\,3}}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0 & dz' \\ x & 0 & -z' \end{vmatrix} = (0, x\,dz', 0)$$



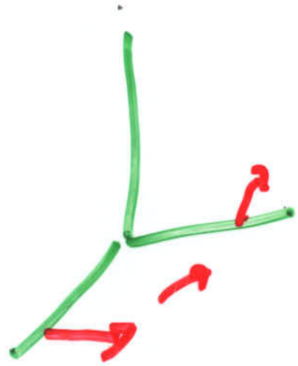
$$B_y = \frac{\mu_0 I}{4\pi} \int \frac{x\, dz'}{\sqrt{\underset{a}{x^2} + \underset{x}{z'^2}}^{\,3}}$$

Dwight 200.03 $\displaystyle\int \frac{dx}{\sqrt{x^2 + a^2}^{\,3}} = \frac{1}{a^2} \frac{x}{r}$

$$B_y = \frac{\mu_0 I}{4\pi}\, x \; \frac{1}{x^2} \frac{z'}{\sqrt{x^2 + z'^2}} \Bigg|_{z' = -\infty}^{z' = +\infty}$$

$\hookrightarrow 1 - -1 = 2$

$$B_y = \frac{\mu_0 I}{4\pi}\frac{1}{y}\,2 = \frac{\mu_0 I}{2\pi y}$$



$$B_\phi = \frac{\mu_0 I}{2\pi r}$$

$(0,0,1)$

$K\hat{k} =$

$$\vec{K}\,dq = \vec{K}\,R\,d\theta\,dz'$$

$$I = K\,2\pi R$$

$$\vec{r} = \langle x, 0, 0\rangle$$

$$\vec{r}\,' = \langle R\underbrace{\cos\theta}_{c}, R\underbrace{\sin\theta}_{s}, z'\rangle$$

$$J = \frac{I}{\text{Area}}$$

$$K = \frac{I}{\text{Length}}$$

$$\vec{r}-\vec{r}\,' = \langle x - Rc, -Rs, -z'\rangle$$

$$|\vec{r}-\vec{r}\,'| = \sqrt{x^2 - 2Rxc + R^2 + z'^2}$$

$$B = \frac{\mu_0}{4\pi}\int \frac{\vec{K}\,dq \times (\vec{r}-\vec{r}\,')}{|r-r'|^3}$$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ x-Rc & -Rs & -z' \end{vmatrix} = (Rs, x-Rc, 0)$$

$$\vec{B} = \frac{\mu_0}{4\pi} \underset{\uparrow}{K} R \int_{-\pi}^{\pi} \frac{(Rs,\ x-Rc,\ 0)\, d\theta\, dz'}{\sqrt{\underbrace{x^2 - 2xRc + R^2}_{a^2} + z'^2}^{\,3}}$$

$$K = \frac{I}{2\pi R}$$

$$B_y = \frac{\mu_0}{4\pi} K R \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} \frac{x - Rc}{\sqrt{\quad}^{\,3}}\, d\theta\, dz'$$

$$= \frac{\mu_0}{4\pi} K R \int_{-\pi}^{\pi} \frac{x - Rc}{x^2 - 2xRc + R^2}\, d\theta\ 2$$

$$= \begin{cases} 0 & x < R \\ \frac{\mu_0 I}{2\pi x} & x > R \end{cases}$$ hope

Definite

Dwight 859.124

$$\int_{-\pi}^{\pi} \frac{(a - b\cos\theta)}{a^2 - 2ab\cos\theta + b^2} = \begin{cases} \frac{2\pi}{a} & a > b \\ 0 & a < b \end{cases}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\pi/\alpha}$$ Gaussian

I^2

$I = \int_0^\infty e^{-\alpha x^2}\,dx \qquad I = \int_0^\infty e^{-\alpha y^2}\,dy$

$$I^2 = \int_0^\infty \int_0^\infty e^{-\alpha(x^2+y^2)} \underbrace{dx\,dy}_{r\,d\theta\,dr}$$

$\theta : 0 \to \pi/2$

$r : 0 \to \infty$

$$I^2 = \frac{1}{2\alpha}\int e^{-\alpha r^2}\, r\,dr\,\underbrace{d\theta}_{\to\ \pi/2}$$

2α

$$= \frac{1}{2\alpha}\int e^{-u}\,du \quad \pi/2$$

$$= \frac{1}{2\alpha} \; -e^{-u}\Big|_0^\infty \quad \pi/2$$

$$= \frac{\pi}{4\alpha}\, 1$$

$$I = \sqrt{\frac{\pi}{4\alpha}} = \frac{1}{2}\sqrt{\pi/\alpha}$$

Complex Variable : Residue ****

$$B_y = \frac{\mu_0 K l z}{4\pi} \int_{-\pi}^{\pi} \frac{x - Rc}{x^2 - 2xRc + R^2}\, d\theta \quad 2$$

$$\int_{-\pi}^{\pi} \frac{(a - b\cos\theta)}{a^2 - 2ab\cos\theta + b^2} = \begin{cases} 2\pi/a & a > b \\ 0 & a < b \end{cases}$$

$x \to a$

$R \to b$

$$= \frac{\mu_0 \frac{I}{2\pi R} R}{4\pi} \quad \frac{2\pi}{x} \quad 2$$

$$\underbrace{\mu_0 \frac{N}{\ell} I}_{K}$$

$B = 0$

$$= \frac{\mu_0 I}{2\pi x}$$

$\vec{r} = (x, 0, 0)$

$\vec{r}' = (Rc, Rs, z')$

$I d\vec{\ell} \to K \hat{\theta}$

$= K(-s, c, 0)$

$\hat{\theta} = (-s, c, 0)$

$\hat{\theta} \cdot \hat{r} = 0 \qquad \hat{\theta} \cdot \hat{x} =$

$(c, s, 0)$

$$\begin{matrix} I\,d\vec{l} \\ \vec{K}\,d\tau \end{matrix} \times (\vec{r}-\vec{r}\,') : \begin{vmatrix} i & j & k \\ -s & c & 0 \\ x-Rc & -Rs & -z' \end{vmatrix}$$

$$= \left(-cz',\ -sz',\ \begin{matrix} Rs^2 - c(x-Rc) \\ R - xc \end{matrix}\right)$$

$$\vec{B} = \frac{\mu_0}{4\pi} K \int \frac{(-cz',\ -sz',\ R-xc)\,R\,d\theta\,dz'}{\sqrt{z'^2 + \underbrace{R^2 - 2xRc + x^2}_{a^2}}^{\,3}}$$

$$B_z = \frac{\mu_0 K R}{4\pi} \int_{-\pi}^{\pi} \frac{R - xc}{R^2 - 2xRc + x^2}\, 2\, d\theta$$

$$= \begin{cases} x < R & B = \mu_0 K \ \checkmark \\ x > R & 0 \end{cases}$$