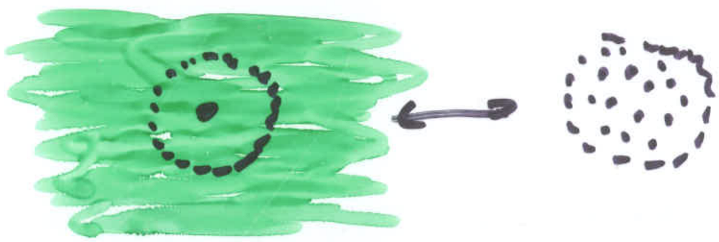
In general "macroscopic" $E \neq E$ felt by atom
$$\overset{\parallel\!\!\mid}{E_m}$$

$$E_m = E - \left(\frac{-P}{3\varepsilon_0}\right) + \sum E_i$$

remove effect of local sphere of continuous dielectric

add back in E due to near by atoms
[0 in simple cases]
cubic, liquid, gas
beware crystal!



relate atomic polarizability to macroscopic K

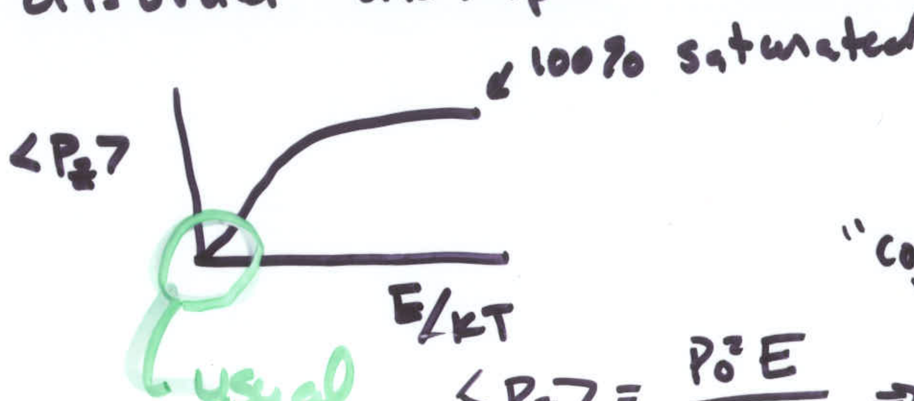
$$\alpha = \frac{3\varepsilon_0 (K-1)}{N(K+2)} = 3\varepsilon_0 V \quad \text{atomic "volume"}$$

very simple model suggests $\alpha = 3\varepsilon_0 V$

$$\Rightarrow VN = \frac{(K-1)}{(K+2)} = \text{fraction of space that is "atom"}$$

polar molecules (ie with dipole) : $PE = -\vec{p}\cdot\vec{E}$
so "downhill" would be $\vec{p} \parallel \vec{E}$ but thermal disorder disrupts $\Delta G = \Delta H - T\Delta S$; $e^{-E/KT}$

ferroelectric
hysteresis
Curie Temp
"condensed Matter Physics"
"solid state"



$$\langle P_z \rangle = \frac{P_0^2 E}{3KT} \Rightarrow \alpha = \frac{P_0^2}{3KT}$$

Energy → in fields E D

not position

→

point charges

$\cdot q_3$

$\cdot q_2$

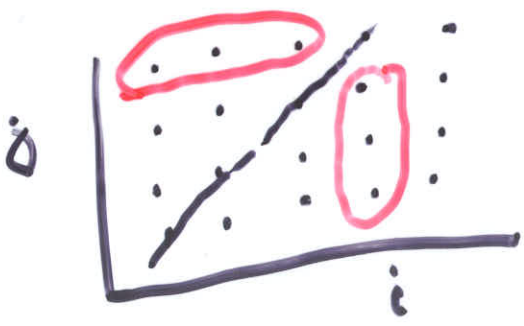
$\cdot q_1$

$U = q_2 V$

$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}}$

new $q_3 V = q_3 \left(\frac{q_1}{4\pi\epsilon_0 r_{13}} + \frac{q_2}{4\pi\epsilon_0 r_{23}} \right)$

$$\text{total } U = \sum_{i=1}^{N} \sum_{j<i} \underbrace{\frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}}_{i \leftrightarrow j}$$



$$U = \frac{1}{2} \sum_i {\sum_j}' \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

$\vec{F} = -\nabla U$

continuous charge $\rho(\vec{r})$ U?

$\nabla \cdot D = \rho$

$\nabla \cdot \delta \bar{D} = \delta\rho$

$\vec{\nabla} \cdot \delta\bar{D}$ $\quad \delta\rho$

$$\delta u = \int \delta\rho\, \phi\, dv$$

$\lambda : 0 \to 1$

$\partial\rho = \rho\, d\lambda$ $\quad \frac{\lambda^2}{2}\Big|_0^1 = \frac{1}{2}$

$= \lambda\phi$

$$= \int_{\lambda} \int_{dv} \rho\, d\lambda\, \lambda\, \phi\, dv$$

$$= \frac{1}{2} \int \rho\phi\, dv$$

$$\delta u = \int \nabla \cdot \delta\bar{D} \cdot \phi\, dv$$

$$\vec{\nabla} \cdot (\vec{A}\phi) = \underline{(\nabla \cdot A)\phi} + \vec{A} \cdot \nabla\phi$$

$$= \int \left(\vec{\nabla} \cdot (\delta D \phi) - \delta D \cdot \nabla\phi\right) dv$$

$$= \int (\delta\bar{D}) \cdot \bar{E}\, dv \qquad D = \varepsilon E$$

$D \propto E$

$$\int \varepsilon\, \delta\bar{E} \cdot \bar{E} \to \delta\left(\tfrac{1}{2} E^2\right)$$

$$\delta u = \int \frac{1}{2}\,\delta(\bar{D}\cdot\bar{E})\,dV$$

$$u = \underbrace{\frac{1}{2}\int \bar{D}\cdot\bar{E}}_{\frac{\text{energy}}{\text{volume}}}\,dV$$

Caps!

$$Q = CV$$

$$\varepsilon_0 K E = D = \sigma = \frac{Q}{A}$$

$$E = \frac{Q/A}{\varepsilon_0 K}$$

$$\Delta V = E\,\Delta x = \frac{d/A}{\varepsilon_0 K}\,Q$$

$$C = \frac{\varepsilon_0 K A}{d}$$

R, ρ find U

$$\frac{1}{2}\int \rho\,\phi\, dV \;;\; \frac{1}{2}\int \vec{E}\cdot\underbrace{\vec{D}}_{\varepsilon_0 E}\, dV$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_0}$$

outside

inside

$$Q\,\frac{r^3}{R^3}$$

$$Q = \rho\,\frac{4}{3}\pi R^3$$

Out: $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

inside $E = \dfrac{Q}{4\pi\varepsilon_0}\,\dfrac{r}{R^3}$

$$U = \frac{\varepsilon_0}{2}\int E^2\, \underbrace{dV}_{4\pi r^2 dr}$$

$$= \frac{\varepsilon_0}{2}\left[\int_0^R \left[\frac{Q}{4\pi\varepsilon_0}\,\frac{r}{R^3}\right]^2 + \int_R^\infty \left[\frac{Q}{4\pi\varepsilon_0 r^2}\right]^2\right]$$

$$= \frac{Q^2}{2\varepsilon_0 4\pi}\left[\int_0^R \frac{r^2\, r^2 dr}{R^6} + \int_R^\infty \frac{1}{r^4}\, r^2 dr\right]$$

$$\frac{1}{5}\,\frac{1}{R} + \frac{1}{R} = \frac{6}{5}\,\frac{1}{R}$$

$$= \frac{3}{5}\,\frac{Q^2}{4\pi\varepsilon_0 R}$$

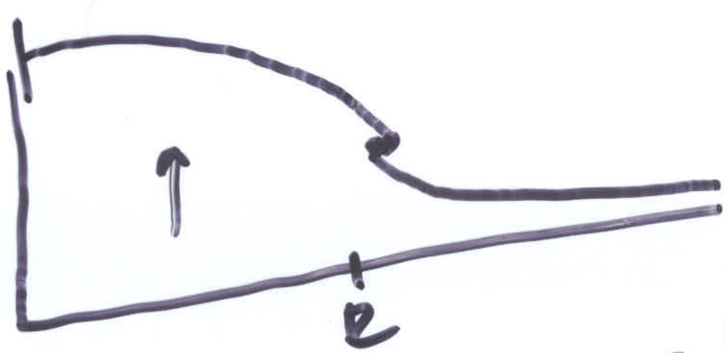
$$E = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0}\dfrac{r}{R^3} \longrightarrow \phi = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{R^3}\left(-\dfrac{1}{2}r^2\right) + C \\[2ex] \dfrac{Q}{4\pi\varepsilon_0}\dfrac{1}{r^2} \rightarrow \phi = \dfrac{Q}{4\pi\varepsilon_0 r} \end{cases}$$

$$\frac{Q}{4\pi\varepsilon_0}\frac{1}{R^3}\left(C - \frac{1}{2}r^2\right)\Bigg|_{r=R} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}\Bigg|_{r=R}$$

$$C - \frac{1}{2}R^2 = R^2$$

$$C = \frac{3}{2}R^2$$

$$\phi = \frac{Q}{4\pi\varepsilon_0 R}\left(\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2\right)$$



$$U = \frac{1}{2}\rho\int_0^R \phi \; 4\pi r^2 dr$$

$$\frac{Q}{\frac{4}{3}\pi R^3}$$

$$= \frac{3}{2}\frac{Q}{R^3}\int_0^R \frac{Q}{4\pi\epsilon_0 R}\left[\frac{3}{2}-\frac{1}{2}\left(\frac{r}{R}\right)^2\right] r^2\,dr$$

$$= \frac{3}{2}\frac{Q^2}{4\pi\epsilon_0 R}\frac{1}{R^3}\int_0^R \left[\underbrace{\frac{3}{2}r^2}_{\frac{1}{3}R^3} - \frac{1}{2}\underbrace{\frac{r^4}{R^2}}_{\frac{1}{5}R^3}\right] dr$$

$$\frac{1}{2} - \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$$

$$= \frac{3}{5}\frac{Q^2}{4\pi\epsilon_0 R}$$

$$U = \frac{\epsilon_0}{2}\int E^2\,dV$$

$$[\vec{E}_1 + \vec{E}_2]^2$$

$$E_1^2 + E_2^2 + 2\vec{E}_1\cdot\vec{E}_2$$

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 d^2}$$

$$\int_0^\infty \left(\frac{Q}{4\pi\epsilon_0 r^2}\right)^2 4\pi r^2\,dr$$

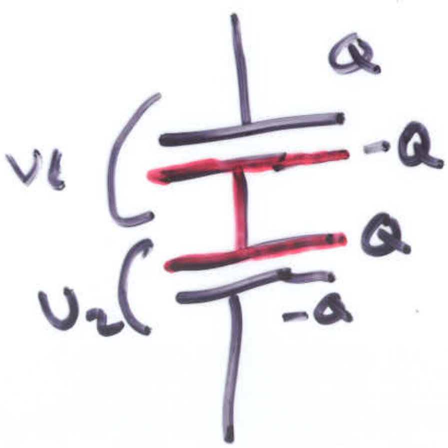
$$\int_0^\infty \frac{1}{r^2}\,dr = -\frac{1}{r}\Big|_0^\infty$$

$$U = \infty + \epsilon_0 \int \vec{E}_1 \cdot \vec{E}_2 \, dV$$

$$U = \frac{1}{2} Q V = \frac{1}{2} C V^2$$

$$Q = C V \qquad = \frac{1}{2} \frac{Q^2}{C}$$

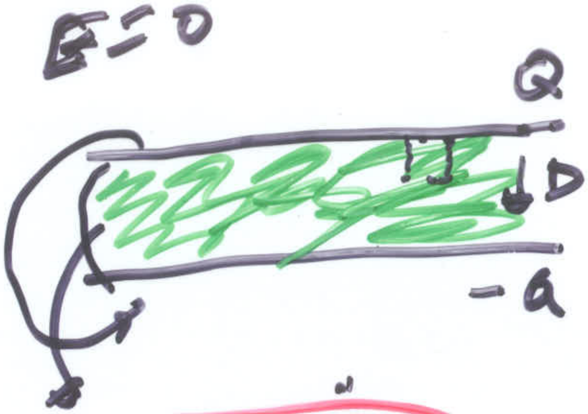
$$Q = Q_1 + Q_2 =$$
$$= C_1 V + C_2 V$$
$$= (\underbrace{C_1 + C_2}_{C_{eq}}) V$$



$$V = V_1 + V_2$$
$$= \frac{Q}{C_1} + \frac{Q}{C_2}$$
$$= \left(\frac{1}{C_1} + \frac{1}{C_2}\right) Q$$

$$\underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}}_{C_{eq}} V = Q$$

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$u = \frac{1}{2}\frac{Q^2}{C}$$

"

$$\frac{1}{2}\int E \cdot D \, dV$$

$dV \to Ad$

$D \to \sigma$

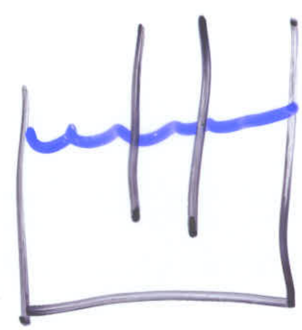
$E \to \frac{D}{\varepsilon_0 \kappa}$

$$C = \frac{\varepsilon_0 \kappa A}{d}$$

$$u = \frac{1}{2}\overbrace{\frac{\sigma}{\varepsilon_0 \kappa}\,\sigma}^{Q/A} A\, d$$

$$= \frac{1}{2}\frac{Q^2\, d}{\varepsilon_0 \kappa A}$$

$$F = {}^{+\partial x}_{-\partial x}\left(\quad\right)$$



$$u = \frac{1}{2} C V^2 \quad \checkmark$$

$$= \frac{1}{2}\frac{Q^2}{C} \quad \checkmark$$

False