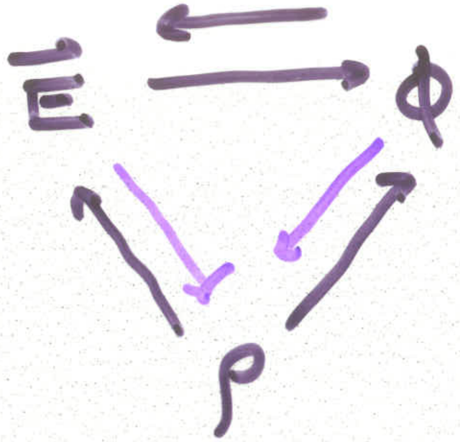


$$\nabla \frac{1}{|\vec{r}-r'|} = \frac{-(\vec{r}-r')}{|\vec{r}-r'|^3}$$

$$\nabla^2 \frac{1}{|\vec{r}-r'|} = 0$$

unless $r=r'$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int \frac{dq'}{|\vec{r}-r'|}$$

dq'
$\rho\, dV'$
$\sigma\, dA'$
$\lambda\, dl'$
points

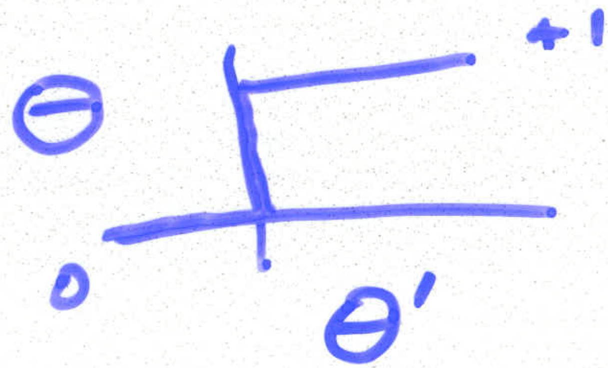
$$\vec{E} = -\nabla\phi$$

$$\phi = \int_r^{GND} \vec{E}\cdot d\vec{l}$$

δ_{ij} Kronecker

δ Dirac $\leftarrow$ $\delta = 0$ if $x \neq x'$

$$\int \delta\, dx = 1$$



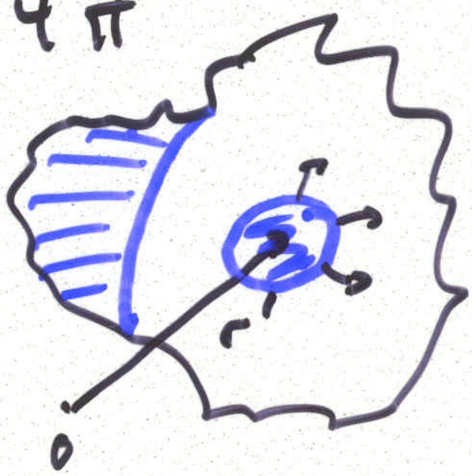
$$\int_{0^-}^{0^+} \Theta'(x)\,dx = \Theta\Big|_{0^-}^{0^+} = (1-0)$$

$\rho \leftrightarrow e^-$ $\quad \int \rho\, dV = -e$

$$\nabla^2 \frac{1}{|\vec{r}-\vec{r}\,'|} = -4\pi\,\delta^3(\vec{r}-\vec{r}\,') \quad \checkmark$$

to show

$$\int \quad dV = -4\pi$$



$$\int \nabla\cdot A = \int \bar{A}\cdot\hat{n}\,dA$$

$$\nabla^2 = \nabla\cdot\nabla\frac{1}{|r-r'|}$$

$$\int \frac{-(\vec{r}-\vec{r}\,')}{|r-r'|^3}\cdot\hat{n}\,dA$$

$$\int \frac{-\hat{n}\cdot\hat{n}}{a^2}\,dA = \frac{-1}{a^2}\overbrace{\int dA}^{4\pi a^2} = -4\pi$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \rho(r')\, dv' \frac{(\vec{r-r'})}{|\vec{r-r'}|^3}$$

$$-\nabla \frac{1}{|r-r'|}$$

$$\nabla \cdot E = \frac{-1}{4\pi\varepsilon_0} \int \rho(r')\, dv' \underbrace{\nabla^2 \frac{1}{|r-r'|}}_{-4\pi\delta(r-r')}$$

$$\int \underbrace{f(x)\,\delta(x-a)}_{f(a)\,\delta(x-a)} dx$$

$$f(a) \int \delta(x-a)\,dx = f(a)$$

$$\int f(x)\,\delta(x-a)\,dx = f(a)$$

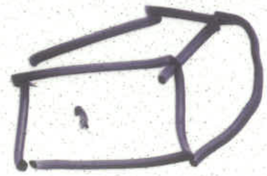
$$\begin{cases} \nabla \cdot E = \dfrac{-1}{4\pi\varepsilon_0} \, -4\pi\rho(r) = \dfrac{\rho}{\varepsilon_0} \, * \\ \nabla \times E = \nabla \times (-\nabla\phi) = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ \partial_x\phi & \partial_y\phi & \partial_z\phi \end{vmatrix} = 0 \end{cases}$$

$$\nabla \cdot \underset{\substack{\uparrow \\ -\nabla\phi}}{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \nabla^2 \phi = \frac{-\rho}{\varepsilon_0}$$

Poisson's Eq

$$\nabla^2 \phi = 0 \checkmark$$

Laplaces



Gauss's Law

$$\int \vec{E} \cdot \hat{n} \, dA = \frac{Q}{\varepsilon_0}$$

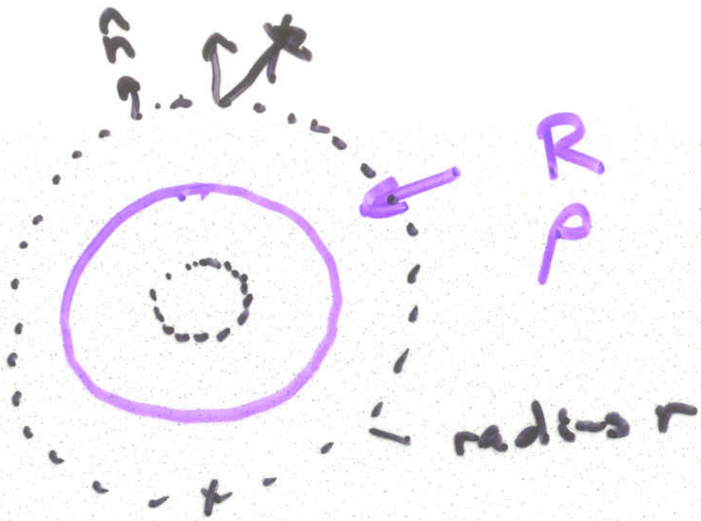
$$\int \underbrace{\nabla \cdot E}_{\rho/\varepsilon_0} \, dV = \int \overbrace{\vec{E} \cdot \hat{n}} \, dA = \phi_E$$

EA

electric flux

$$\rho/\varepsilon_0$$

$$= \frac{Q}{\varepsilon_0}$$



$$\vec{E} = E(r)\,\hat{r}$$

$$\int_r^{GND} E\cdot dl$$

$$\int \vec{E}\cdot\hat{n}\,dA = E(r)\int dA = E\,4\pi r^2$$

$$= \frac{Q}{\varepsilon_0} = \frac{\frac{4}{3}\pi R^3\rho}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = -\nabla\phi = -\partial_r\phi$$

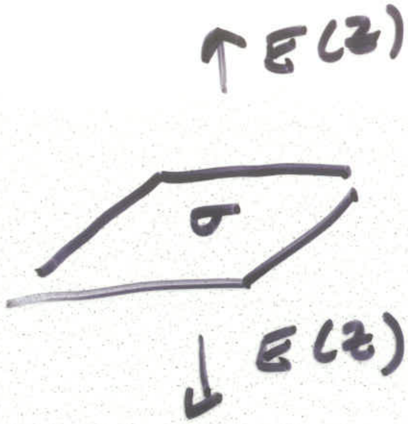
$$\frac{\frac{4}{3}\cancel{\pi}\, r^{\cancel{3}}\rho}{\varepsilon_0} = E\,\cancel{4\pi}\, r^{\cancel{2}}$$

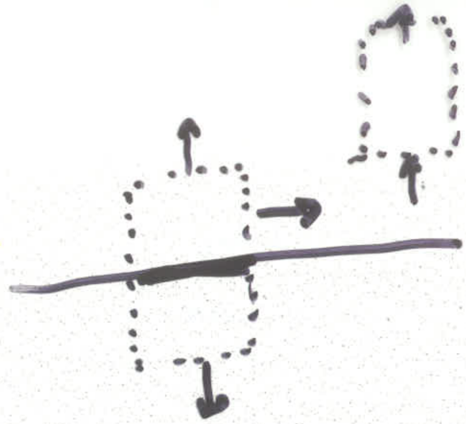
$$\frac{\rho\, r}{3\varepsilon_0} = E$$

$$\phi = \frac{\rho}{3\varepsilon_0}\,{-\tfrac{1}{2}r^2} + C$$

$$\phi = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\frac{Q}{4\pi\varepsilon_0 R} = \phi(R) = C - \frac{\rho}{6\varepsilon_0}R^2$$



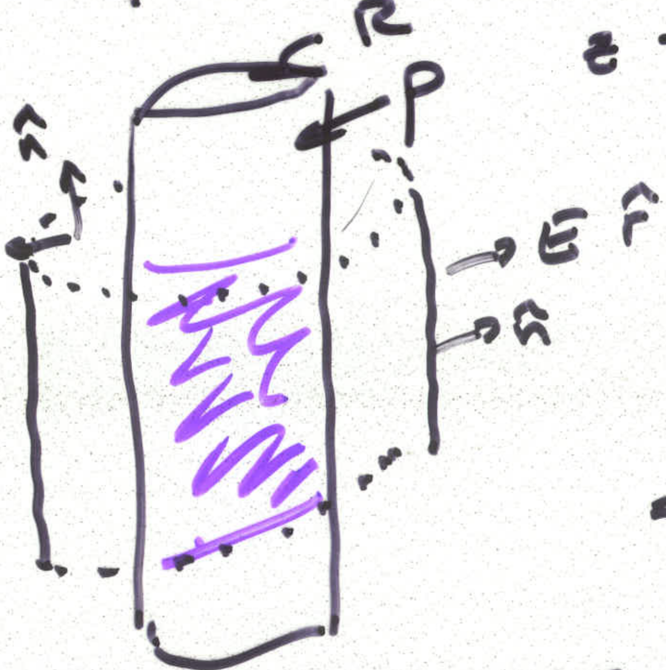


$$\oint \vec{E}\cdot\hat{n}\, dA = A_t E_t(z) + A_b E_b(z)$$
$$= 2E(z)A = \frac{\sigma A}{\varepsilon_0}$$

$$E(z) = \frac{\sigma}{2\varepsilon_0}$$

$$\phi(z) = -\frac{\sigma}{2\varepsilon_0}|z|$$

$z \to \pm\infty$



$E(r)$

$$\oint E(r)\,\hat{r}\,\hat{n}\, dA$$
$$= E(r)\underbrace{A}_{2\pi r l}$$
$$= \frac{Q}{\varepsilon_0} = \frac{\rho \pi R^2 l}{\varepsilon_0}$$

$$E\, 2\cancel{\pi} \cancel{r} \cancel{\ell} = \frac{\rho \cancel{\pi} R^2 \cancel{\ell}}{\varepsilon_0}$$

$$E = \frac{\rho R^2}{2 \varepsilon_0 r}$$

$$\phi = \frac{-\rho R^2}{2\varepsilon_0} \ln(r)$$

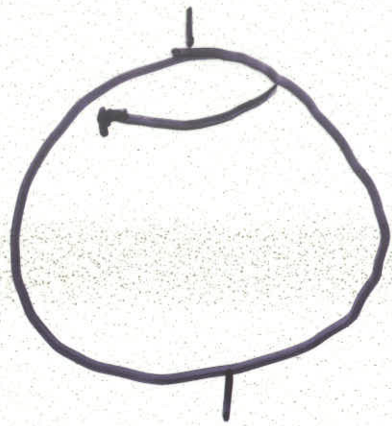
$$\phi = \frac{-\rho R^2}{2\varepsilon_0} \ln\left(\frac{r}{R}\right)$$

$E \qquad \phi$

$\rho \leftarrow V$

$V(\theta, \phi)$

$V(\theta)$



$\rho = 0$

$\nabla^2 \phi = 0$

$\sum a_n X(x) Y(y)$

$\partial_x^2 \phi + \partial_y^2 \phi = 0$

$\phi(x,y) = X(x)\, Y(y)$

$x^2 + y^2$

Fouriers Trick

$$\partial_x^2 \phi + \partial_y^2 \phi = 0 \qquad \phi = XY$$

$$YX'' + XY'' = 0$$

$$XY'' = -YX''$$

$$\frac{1}{XY} \qquad \frac{Y''}{Y} = \frac{-X''}{X} = -m^2$$

$$\underline{\text{const}} = \frac{Y''}{Y}$$

$$X'' = m^2 X$$

$$Y'' = -m^2 Y$$

$$X = e^{\pm mx}$$

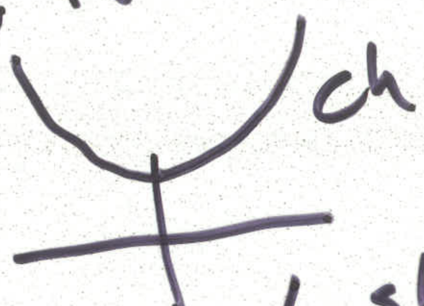
$$= \cosh mx, \ \sinh mx$$

$$Y = \sin(my)$$

$$\cos$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler



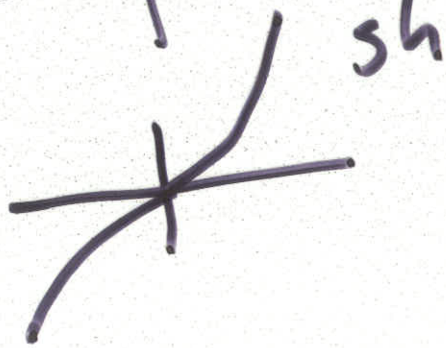
$$e^x =$$

$$i^2 = -1$$

$$\cos x =$$

$$\sin x =$$

$$j^2$$



$$f(x_0+\Delta x) = f(x_0) + f'(x_0)\,\Delta x$$

$$+ \frac{1}{2!} f''(x_0)\,\Delta x^2 + \frac{1}{3!}$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots + \frac{1}{4!}x^4$$

$$\cos x = 1 - \frac{1}{2}x^2 +$$

$$\sin x = x - \frac{1}{3!}x^3$$

$\cos^2 + \sin^2 = 1$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$\cosh^2 - \sinh^2 = 1$