Class 3   old exam #1, 3.21, 3.22, I square, I shell

#1 a) $M = 1+4 = 5$ $\vec{V}_{cm} = \frac{-4\hat{j}}{5} = -\frac{4}{5}\hat{j}$

$$\vec{R}_{cm} = \frac{(-1,4)\cdot 1 + (-2,0)\cdot 4}{5} = \frac{(-9,\ 4)}{5}$$

b) $\vec{l}_2 = 0$, $\quad l_1 = 1\begin{vmatrix} i & j & k \\ -1 & 4 & 0 \\ 0 & -4 & 0 \end{vmatrix} = 4\hat{k}$

final: $\vec{V}_{cm} = \frac{1}{5}\left[\left(\frac{16}{5}, \frac{-4}{5}\right) + 4\left(\frac{-4}{5}, \frac{-4}{5}\right)\right] = \frac{1}{5}\left(0, \frac{-20}{5}\right) = \left(0, \frac{-4}{5}\right)$ unchange

$\vec{R}_{cm} = \frac{1}{5}\left[(7,-1) + 4(-4,-2)\right] = \frac{1}{5}(-9,-9) = \left(-\frac{9}{5}, -\frac{9}{5}\right)$

unchanged as $V_{cm\,x} = 0$

$$L_{orbit} = 5\begin{vmatrix} i & j & k \\ -9/5 & -9/5 & 0 \\ 0 & -\frac{4}{5} & 0 \end{vmatrix} = \frac{36}{5}\hat{k}$$

$$L_{spin} = \underset{4/5}{\mu}\,\underset{(11,1)}{(\vec{r}_1 - \vec{r}_2)} \times \underset{(4,0)}{(v_1 - v_2)} = \frac{4}{5}\begin{vmatrix} i & j & k \\ 11 & 1 & 0 \\ 4 & 0 & 0 \end{vmatrix} = \frac{-16}{5}$$

Note $L_{orbit} + L_{spin} = \frac{36-16}{5} = \frac{20}{5} = 4\hat{k}$ ← equals initial L

c) $T = \underbrace{\frac{1}{2}MV_{cm}^2}_{\text{same}} + \frac{1}{2}\mu\underbrace{(v_1 - v_2)^2}_{\text{same (direction changed but not magnitude)}} = \text{same!}$

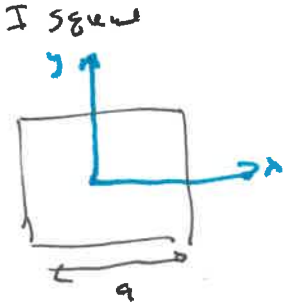
d) see above pre & post same $= \frac{1}{2}\cdot 1\cdot 4^2 = 8$

check: $\frac{1}{2}5\cdot\left(\frac{4}{5}\right)^2 + \frac{1}{2}\frac{4}{5}\cdot 4^2$ or: in initial all KE is in 1

$= \frac{8}{5} + \frac{32}{5} = \frac{40}{5} = 8$ ✓ $\frac{1}{2}\cdot 1\cdot 4^2 = 8$ ✓

"OF"   "ABOUT"

I square



$$I = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} \sigma\, dx\,dy\,(x^2+y^2) = \frac{M}{a^2}\int_{-a/2}^{a/2}\left(\frac{x^3}{3} + y^2 x\right)\Big|_{-a/2}^{a/2} dy$$

$\sigma = \frac{\text{mass}}{\text{area}} = \frac{M}{a^2}$

$$= \frac{M}{a^2}\left(a^3\frac{1}{12}y + \frac{1}{3}y^3 a\right)\Big|_{-a/2}^{a/2}$$

$$= \frac{M}{a^2}\left(\frac{1}{12} + \frac{1}{12}\right)a^4$$

$$= \frac{1}{6}Ma^2$$

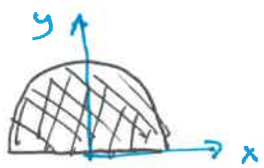
I spherical shell $\rightarrow$ $\frac{2}{5}MR^2 = \frac{2}{5}\rho\frac{4}{3}\pi R^5$

$$\frac{8}{15}\rho\left[(R+\Delta R)^5 - R^5\right] \approx \frac{8}{15}\rho\left[5R^4\Delta R\right]\cancel{\pi} = \frac{2}{3}\left[R^2\right]\underbrace{4\pi R^2\Delta R\rho}_{M}$$

M shell $= 4\pi R^2 \Delta R\,\rho$

$$= \frac{2}{3}MR^2$$

3.21) By symmetry CM must be on y axis



$2x = 2\sqrt{R^2-y^2}$ $\;dy$

$$R_{cm} = \frac{1}{M}\int_0^R 2\sqrt{R^2-y^2}\,dy\;y\,\sigma$$

$\sigma \cdot \frac{1}{2}\pi R^2$

$$= \frac{1}{M}\int_0^R \underset{u}{\sqrt{R^2-y^2}}\;\underset{du}{2y\,dy}\;\sigma$$

$$= \frac{1}{M}\int_0^{R^2}\sqrt{R^2-u}\,du\;\sigma$$

$$= \frac{1}{M}\left.\frac{-(R^2-u)^{3/2}}{3/2}\right|_0^{R^2}\sigma$$

$$= \frac{1}{M}\frac{2}{3}R^3\sigma$$

$$\frac{1}{\frac{1}{2}\pi R^2}\frac{2}{3}R^3 = \frac{4}{3\pi}R$$

ALT: $$\frac{1}{M}\int_0^{\pi}\int_0^R \underbrace{r\,dr\,d\theta}_{dArea}\;r\sin\theta\;\sigma$$

$$= \frac{\sigma}{M}\int_0^R r^2\,dr\;\underset{2}{\left.(-\cos\theta)\right|_0^{\pi}}$$

$$= \frac{\sigma}{M}\frac{2}{3}R^3 \checkmark$$

3.22: By symmetry CM on z axis – divide into disks

$dV = \pi r^2 dz$, $\;r^2 = R^2 - z^2$

$$R_{cm} = \frac{1}{M}\int_0^R \pi(R^2-z^2)\,dz\,\rho\,z$$

$$= \frac{\pi\rho}{M}\left[\frac{1}{2}R^2z^2 - \frac{1}{4}z^4\right]_0^R$$

$$= \frac{\pi\rho}{M}\left[\frac{1}{4}R^4\right] \qquad M = \frac{2}{3}\pi R^3\rho$$

$$= \frac{3}{8}R$$

ALT: $dV = r^2 dr \sin\theta\, d\theta\, d\phi$

$$R_{cm} = \frac{1}{M}\iiint r^2 dr \sin\theta\, d\theta\, d\phi\; r\cos\theta\,\rho$$

$$= \frac{2\pi\rho}{M}\int_0^R\int_0^{\pi/2} r^3 dr\,\sin\theta\cos\theta\,d\theta$$

Let $\cos\theta = c$

$$= \frac{2\pi\rho}{M}\int_0^R r^3 dr\,\left.\frac{c^2}{2}\right|_0^1$$

$$= \frac{\pi\rho}{M}\frac{R^4}{4}$$

$\checkmark$