Class 2 - old exam #4, 5.8, 5.13, SHO3

#4) $\tau \dot{x} = -x$

$$-\frac{dx}{x} = \frac{dt}{\tau}$$

$$\ln\left(\frac{x}{x_0}\right) = \ln(x)\Big|_{x_0}^{x} = -\frac{t}{\tau}$$

$$x = x_0 e^{-t/\tau}$$

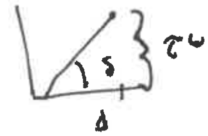
↑ one adj const

$x = Ae^{i\omega t}$

$$(\tau i\omega + 1) A e^{i\omega t} = f_0 e^{i\omega t}$$

$$A = \frac{f_0}{1 + i\tau\omega} = \frac{f_0}{\sqrt{1+(\tau\omega)^2}\, e^{i\delta}}$$

$$= \frac{f_0}{\sqrt{1+(\tau\omega)^2}} e^{-i\delta}$$



$\tan\delta = \omega\tau$

$$|A| = \frac{f_0}{\sqrt{1+(\tau\omega)^2}}$$

(c) f_0 for $\omega = 1$

f_0 for $3\omega = -\frac{1}{3}$

$$X = A(\omega)\cos(\omega t - \delta) - \frac{1}{3} A(3\omega)\cos(3\omega - \delta)$$

$\delta(\omega) = \delta_1 = \tan^{-1}(\omega\tau)$ $\quad$ $\delta(3\omega) = \delta_3 = \tan^{-1}(3\omega\tau)$ ← note

$$= \frac{1}{\sqrt{1+(\tau\omega)^2}}\cos(\omega t - \delta_1) - \frac{1/3}{\sqrt{1+(\tau 3\omega)^2}}\cos(3\omega t - \delta_3)$$

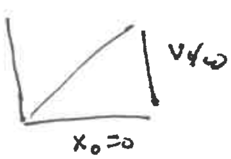
↑ note

5.8 $\omega^2 = \frac{k}{m} = \frac{80}{.2}$; $\omega = \sqrt{\frac{80}{.2}} = 20\ s^{-1}$; $f = \frac{\omega}{2\pi} = 3.18\ Hz$

$T = \frac{1}{f} = .314$ sec

$$A = \sqrt{(x_0)^2 + (v_0/\omega)^2} = \sqrt{\left(\frac{40}{20}\right)^2} = 2\ m$$

δ:



$\rightarrow \delta = \pi/2$ $\quad$ ie $\cos(\omega t - \delta) = \sin(\omega t)$

5.13

$$U' = U_0\left(\frac{1}{R} - \lambda^2 \frac{R}{r^2}\right) = 0 \rightarrow r = \lambda R$$

$$U'' = U_0\left(2\lambda^2 \frac{R}{r^3}\right)\Big|_{\lambda R} = \frac{U_0\, 2}{\lambda R^2}$$

$$\therefore U \approx U\big|_{\lambda R} + \frac{1}{2}\underbrace{\frac{U_0\, 2}{\lambda R^2}}_{K}\Delta r^2$$

$$\omega^2 = \frac{k}{m} = \frac{2U_0}{m\lambda R^2}$$

SHO3: yes A:  ; yes: δ 

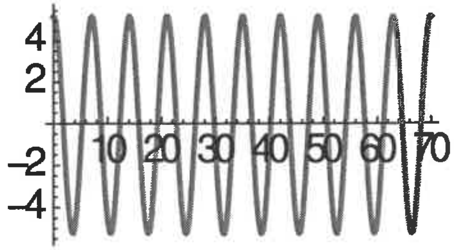
The first 8 plots have $\beta = .01$) — underdamped
$\omega_0 = 1$ so $Q = \frac{\omega_0}{2\beta} = 50$

The bottom 2 have $\beta = 1$ & 2
$\omega_0 = 1$ so $Q = \frac{1}{2}$ & $\frac{1}{4}$

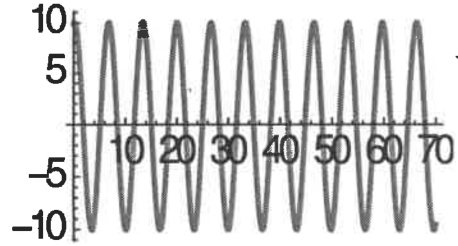
critical damping — over damped

looks like cos below resonance $\delta \sim 0$

$\omega = .9$



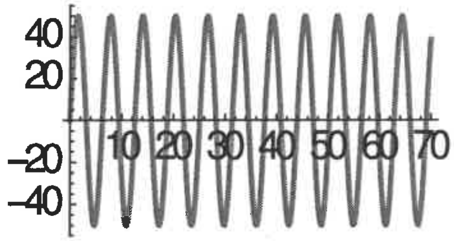
$\omega \approx .95$



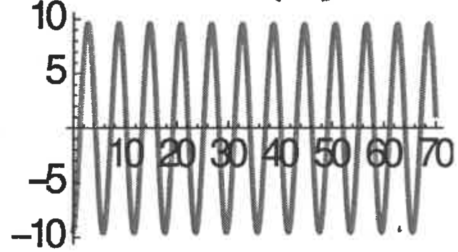
— below resonance

looks like sin $\delta \sim \pi/2$ at resonance →

$\omega = 1$



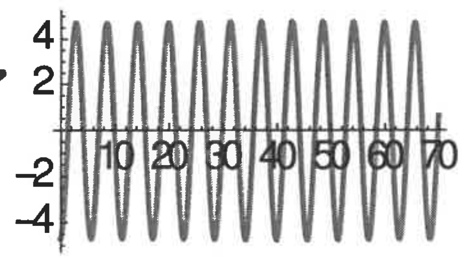
$\omega \approx 1.05$



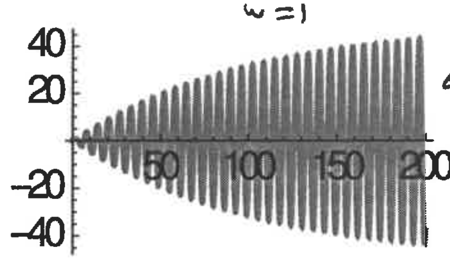
— above resonance

looks like −cos above resonance $\delta \sim \pi$ →

$\omega = 1.1$



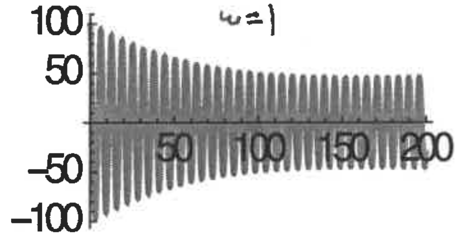
$\omega = 1$



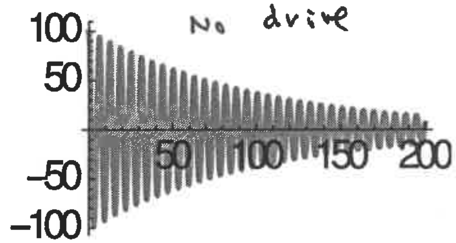
← starts from nothing & builds to equilibrium $A \sim 50$

starts too big decays to resonance →

$\omega = 1$



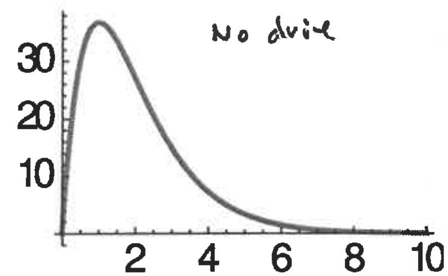
No drive



— underdamped decay

critical damped

No drive



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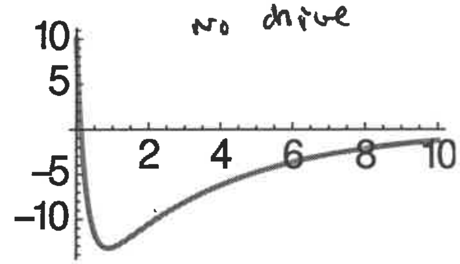


over damped

$WAPP^+$: Fit Results

An analysis of data submitted by computer: on 27-OCT-2020 at 11:30 indicates that a function of the form:

--Resonance-- $1/y^2 = A + Bx^2 + Cx^4$

can fit the 5 data points with a reduced chi-squared of 0.23E-01

PARAMETER	FIT VALUE	ERROR
A =	0.9968	0.20E-01
B =	-1.993	0.40E-01
C =	0.9965	0.20E-01

Resonance Parameters:

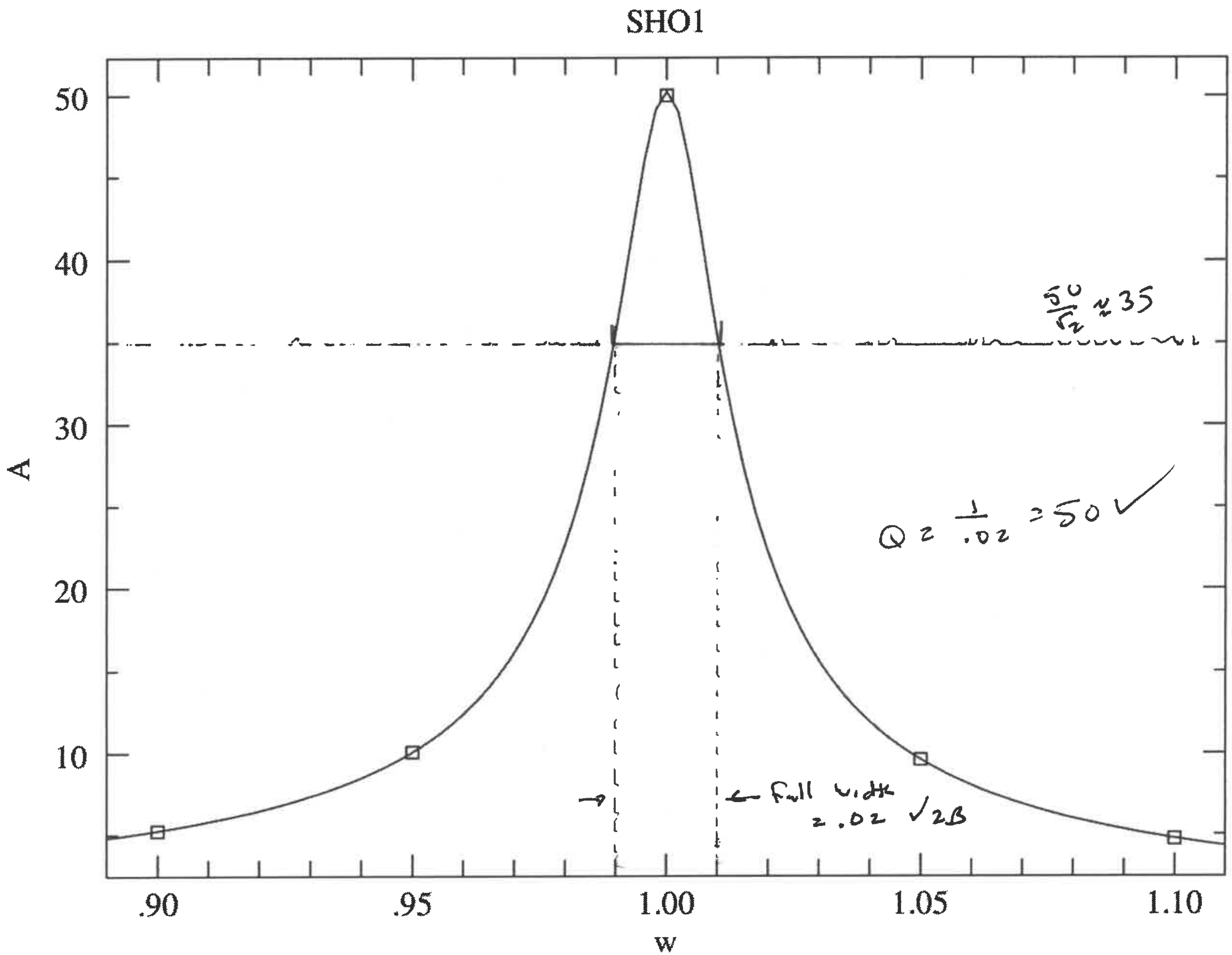
y_0 =	1.002	0.11E-01
f_0 =	1.000	NaN
Q =	50.13	0.58

NO x-errors
NO y-errors

POINT	X	ACTUAL Y	CALCULATED Y	DEVIATION FROM FIT
1	0.900	5.22	5.25	-0.229E-01
2	0.950	10.1	10.1	0.537E-02
3	1.00	50.0	50.2	-0.213
4	1.05	9.59	9.59	0.589E-02
5	1.10	4.72	4.75	-0.249E-01

See spreadsheet
SHO1.

Data Reference: 0896


SHO1
A
50
40
30
20
10
.90
.95
1.00
1.05
1.10
w
$\frac{50}{\sqrt{2}} \approx 35$
$Q = \frac{1}{.02} = 50$
full width
$= .02$