Class 12 : 10.43, I tensor.pdf, coil flip, pencil gyro, B07.25

10.43 : Disk: $\underset{\frac{1}{2}MR^2}{I_3 \overset{\shortparallel}{}} = 2 I_1$ & $I_1 = I_2$

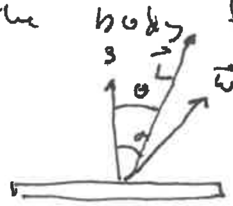
show: $|\vec{\omega}|$ const — clear from 10.94 as $\cos^2\Omega t + \sin^2(\Omega t) = 1$ ← doesn't depend on t.

starting earlier: (10.89)

$$\left.\begin{aligned} \omega_1 \times \quad I_1 \dot{\omega}_1 &= (I_1 - I_3)\,\omega_2 \omega_3 \\ \omega_2 \times \quad I_1 \dot{\omega}_2 &= (I_3 - I_1)\,\omega_1 \omega_3 \end{aligned}\right\} \text{ used } I_1 = I_2$$

$$\underbrace{I_1(\omega_1 \dot{\omega}_1 + \omega_2 \dot{\omega}_2)}_{\frac{d}{dt}(\omega_1^2 + \omega_2^2)} = (I_1 - I_3)(1 - 1)\,\omega_1 \omega_2 \omega_3 = 0$$

In the body frame:



$$\tan\theta = \frac{L_\perp}{L_3} = \frac{I_1 \omega_\perp}{I_3 \omega_3} = \frac{I_1}{I_3}\tan\alpha = \frac{1}{2}\tan\alpha$$

From Eq 12 in "Euler Angles & Free Precession"

$$\dot{\phi} = \frac{I_3 \omega_3}{I_1 \cos\theta} = \frac{2\omega_3}{\cos\theta} \qquad \text{Now} \quad \tan^2\theta + 1 = \frac{1}{\cos^2\theta}$$

$$= 2\omega_3 \sqrt{\tan^2\theta + 1} = 2\omega_3 \sqrt{\left(\tfrac{1}{2}\tan\alpha\right)^2 + 1}$$

$$= \omega_3 \sqrt{\tan^2\alpha + 4} = \omega\cos\alpha \sqrt{\tan^2\alpha + 4}$$

$$= \omega \sqrt{\sin^2\alpha + 4\cos^2\alpha} \leftarrow 1 - \sin^2\alpha$$

$$= \omega \sqrt{4 - 3\sin^2\alpha}$$

Chandler: using google & seeking images find

Earth $I_3 > I_1$ which is ↻ yes ✓

Ref: "Euler Angles & Free Precession"

Physics 339 — Wobble of Symmetric Objects — November 2020

Consider the following three objects all thrown with $\omega_3 = 40$ rad/s but with a small off-axis spin such that $\theta = 10°$

A. A Frisbee of mass $M = 175$ g and radius $R = 13.7$ cm (we ignore the height which is about 3.4 cm)

B. A wooden dowel with $M = 38$ g, radius $R = .94$ cm and length $\ell = 23$ cm

C. A thin-walled cylinder with $M = 16$ g, radius $R = 2.4$ cm and length $\ell = 8$ cm

For a disk:

$$I_A = \frac{1}{4}MR^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \rightarrow \frac{I_3}{I_1} = 2$$

for a solid cylinder:

$$I_B = \frac{1}{12}M \begin{pmatrix} 3R^2+\ell^2 & 0 & 0 \\ 0 & 3R^2+\ell^2 & 0 \\ 0 & 0 & 6R^2 \end{pmatrix} \rightarrow \frac{I_3}{I_1} = \frac{6R^2}{3R^2+\ell^2} = \frac{6 \cdot .94^2}{3 \cdot .94^2 + 23^2} \approx .01$$

as seen from above

$I_3 > I_1 \rightarrow$ cc

$I_3 < I_1 \rightarrow$ clockwise

Eq 13: $\dot\psi = \frac{I_1 - I_3}{I_1}\omega_3$

for a thin-walled cylinder:

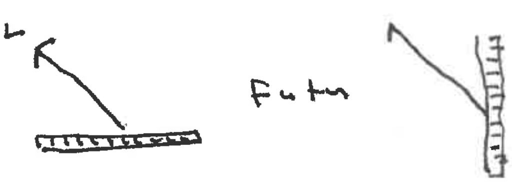
$$\frac{I_3}{I_1} = \frac{12R^2}{6R^2+\ell^2} \approx .7 \leftarrow I_C = \frac{1}{12}M \begin{pmatrix} 6R^2+\ell^2 & 0 & 0 \\ 0 & 6R^2+\ell^2 & 0 \\ 0 & 0 & 12R^2 \end{pmatrix}$$

For each object report the wobble frequency in the body frame (and the direction of $\omega_\perp$ motion: same as or reverse from ω_3) and the wobble frequency in the inertial frame.

Eq 12: $\dot\phi = \frac{I_3\omega_3}{I_1 \cos\theta_0}$

	inertial wobble	body wobble	
disk:	81 rad/sec	−40 rad/sec	cc
solid cylinder	.4	39.6	clockwise
thin cylinder	28.4	12	clockwise

Coin Flip:



requires 45° angle between 3 axis & L

have shown: $\tan\theta = \frac{1}{2}\tan\alpha \rightarrow \frac{\omega_\perp}{\omega_3} = 2$

pencil: $R = .35$ $\ell = 15$ cm ; offset of end from CM $= \ell/2$

$$I_1 = I_2 = \frac{1}{12} M(3R^2 + \ell^2) + M\left(\frac{\ell}{2}\right)^2$$

unchanged $I_3 = \frac{1}{12} M \, 6R^2$

distance to CM $= \ell/2$

in formula for $c^2 = \frac{mgK}{I_1} = \frac{g(\ell/2)}{\frac{1}{12}(3R^2+\ell^2) + \left(\frac{\ell}{2}\right)^2} = \cancel{\quad} \frac{1}{s^2}$

~ 10

$480 \, \mathrm{cm}/\mathrm{s}^2$

Run code with $\dot{\phi}(0) = 0$ find [euler_angles + top.m]

$a = 10 \frac{1}{\sec}$ has $\sim 10°$ nutation.

$P_\psi = I_3 \omega_3 = I_1 a$

$\omega_3 = \frac{I_1}{I_3} a = \cancel{h} \sim 10^4 \frac{1}{\sec} \cdot \frac{60\sec}{\min} \frac{\text{rev}}{2\pi \,\text{rad}} = 10^5 \text{ rpm}$

$$\frac{\frac{1}{12}(3R^2 + \ell^2) + \left(\frac{\ell}{2}\right)^2 \simeq \frac{1}{3}\ell^2}{\frac{1}{2}R^2} = \frac{2}{3}\left(\frac{\ell}{R}\right)^2$$

$$= \frac{2}{3}\left(\frac{15}{.35}\right)^2 \simeq 1.22 \times 10^3$$