Kirchhoff's Laws

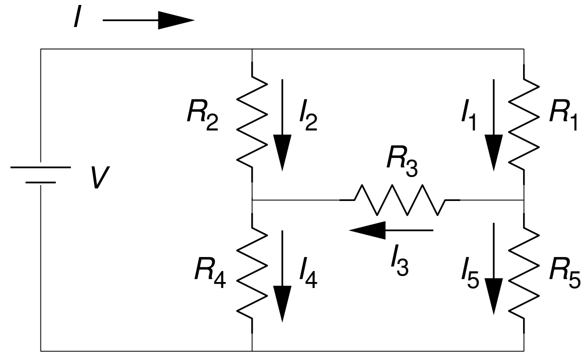


Figure 1: The above circuit was devised by Charles Wheatstone in 1843, and hence is often called a Wheatstone bridge. Generally it is set up so that V, R_2–R_5 are known values, and R_1 is varied until I_3 is zero.

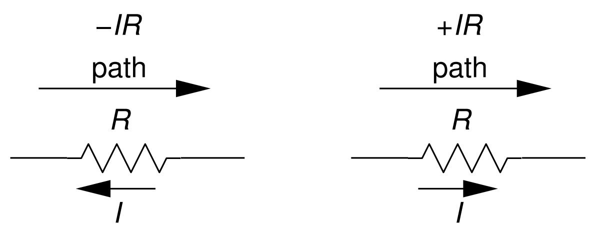
1 Discussion

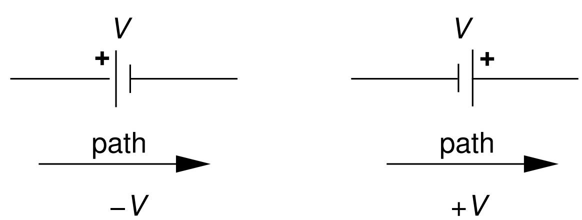
Gustav Kirchhoff (1824–1887) discovered two laws which describe the current flow in dc circuits:

1. The sum of the currents flowing away from any point in a circuit is equal to the sum of the currents flowing toward that point. For example, in the above circuit we must have:

$$I_1 = I_3 + I_5$$

2. In traversing any closed path in a circuit the sum of the emfs (e.g., battery voltages) is equal to the sum of the IR voltage drops from resistors. The sign of the emfs and IR drops is determined by the traversal direction, battery polarity, and assumed direction of I as follows:





For example, in the above circuit one path gives the equation:

$$V = I_2R_2 - I_3R_3 + I_5R_5$$

2 Homework

If we consider V and R_1–R_5 as fixed there are six unknowns: I, I_1–I_5. Write down six independent linear equations relating these unknowns, and solve the resulting system of linear equations with `Solve`. Use *Mathematica* to find an expression for I_3:

```
                                   r1 r4 - r2 r5
Out[4]= --------------------------------------------------------------
        (r3 r4 + r2 (r3 + r4)) r5 + r1 (r4 (r3 + r5) + r2 (r3 + r4 + r5))
```

Show that $I_3 = 0$ only if

$$\frac{R_1}{R_5} = \frac{R_2}{R_4}$$

(Since *Mathematica* likes to reserve capital letters for its own use, I used `r1, r2, ...` for $R_1, R_2, \ldots$)

Turn in a printout showing each step as *Mathematica* solves the problem.