Halley's Comet

1 Discussion

> And now that we have described the system of the Sun, the Earth, Moon, and planets, it remains that we add something about the comets
>
> Isaac Newton (at the conclusion of Proposition XXXIX, Book III)

The People:

Edmond Halley (*c*1656–1743) We remember Halley for "Halley's comet", which he sighted in 1682, and later showed it was a regular, but infrequent, visitor to the inner Solar System. But the greatest event in Halley's life occurred two years later when, on a visit to Cambridge, he happened to ask Newton "what he thought the Curve would be that would be described by the Planets supposing that the force of attraction towards the Sun to be the reciprocal to the square of their distance from it." [1] Newton replied immediately that he had calculated it to be an ellipse. (Both men knew that ellipses were the orbit Kepler had found best matched Tycho's observations of the planets.) Halley convinced Newton to publish his results, and eventually Halley oversaw and financed the publication of Newton's masterpiece.

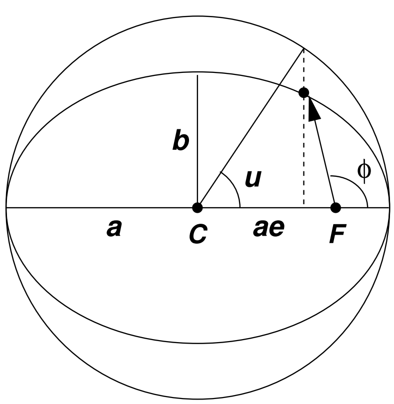
Isaac Newton (1642–1727) The publishing of Newton's *Principia* in **1687** is perhaps the most important event in the history of physics. But we turn to an earlier event that effected the path Newton took to writing his great work: the great comets of 1680. Starting in December of 1680, Newton carefully watched as this monster comet moved through the sky. He apparently constructed a large reflecting telescope to continue his watch as the comet dimmed. He consulted Halley on observations and disputed with Royal Astronomer John Flamsteed as to the motion of this comet(s?). It may well be that he first realized that the force of gravity extended to comets at this time. Calculating the orbit of the comet of 1680 was a major task for publication of Book III of the *Principia*. It is ironic that in proving that comets were not supernatural, Newton provided a great example of how comets are portents of great events in history.

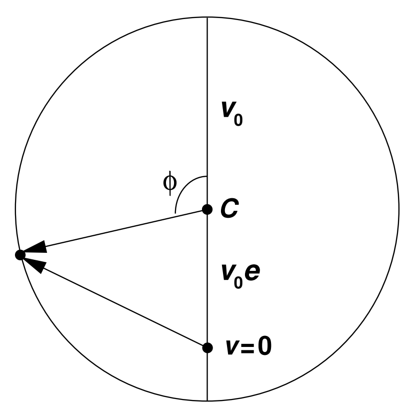
The Problem:

Orbits are complicated things, and multi-media helps explain them. I've collect background material in:

`http://www.physics.csbsju.edu/orbit/`

`orbit.2d.html` describes the shape of the orbit and the timing of the motion. `orbit.3d.html` describes how the orientation of the orbital plane is described. Please read this material on-line. Below I record some of the equations and diagrams without explanation. Note that the constant α is GMm where G is Newton's gravitational constant, M is the mass of the Sun, and m is the mass of the comet.





[1] Newton's recollection as he told it to Abraham DeMoivre

$$\mathbf{a} = \frac{d}{dt}\,\mathbf{v}(t) = \frac{1}{m}\,\mathbf{F}(\mathbf{r}(t)) = -\,\frac{\alpha\hat{\mathbf{r}}}{mr^2}$$

$$b = a\sqrt{1-e^2}$$

$$\mathbf{r} = (a(\cos u - e), b\sin u)$$

$$\tan\left(\frac{\phi}{2}\right) = \sqrt{\frac{1+e}{1-e}}\,\tan\left(\frac{u}{2}\right)$$

$$u - e\sin u = \omega t \qquad \text{where: } \omega = \sqrt{\frac{\alpha}{ma^3}}$$

$$v_0 = \sqrt{\frac{\alpha}{ma(1-e^2)}}$$

$$|\mathbf{a}| = v_0\dot{\phi}$$

$$L = mr^2\dot{\phi} = mv_0(1+e)\,a(1-e) = \sqrt{\alpha ma(1-e^2)}$$

$$\begin{aligned} |\mathbf{a}| &= v_0\dot{\phi} \\ &= v_0\left(\frac{v_0 a(1-e^2)}{r^2}\right) \\ &= \frac{\alpha}{mr^2} \end{aligned}$$

$$E = -\,\frac{\alpha}{2a}$$

$$V_{\text{eff}}(r) = U(r) + \frac{L^2}{2mr^2}$$

The following section shows *Mathematica* making 2d plots of the orbit and hodograph for $a = 1$ and $e = .707$. A note on units: distances in the Solar System are measured in astronomical units: AU, the mean distance between Earth and Sun.

```
a=1; e=N[1/Sqrt[2]]

uu=Table[0,{51}]
rr=Table[{0,0},{51}]

Do[
uu[[i]]= u /. FindRoot[u-e Sin[u] == .04 Pi (i-26),{u,.04 Pi (i-26) }] ;
rr[[i,1]]=a (Cos[uu[[i]]]-e) ;
rr[[i,2]]=a Sqrt[1-e^2] Sin[uu[[i]]] ,
{i,51}]

ListPlot[rr]

ParametricPlot[{a (Cos[u]-e),a Sqrt[1-e^2] Sin[u ]},{u,-Pi,Pi}]
Show[%%,%,AspectRatio->Automatic]

pp=Table[0,{51}]
Do[
pp[[i]]=N[2 ArcTan[ Cos[uu[[i]]/2], Sqrt[(1+e)/(1-e)] Sin[uu[[i]]/2] ] ],
{i,51}]
vxy=Table[{-Sin[pp[[i]]],Cos[pp[[i]]]+e},{i,51}]
ListPlot[%,AspectRatio->Automatic]
ParametricPlot[{-Sin[u ],(Cos[u]+e) },{u,-Pi,Pi}]
Show[%,%%,AspectRatio->Automatic]
```

The following section shows *Mathematica* comparing the orbit of Jupiter to that of Halley's comet.

```
<<Geometry/Rotations.m
a= 17.94; e=.9673; omega=N[111.8 Pi/180]; nodes=N[58.1 Pi/180]; inclin=N[162.2 Pi/180];
rr=Table[{0,0,0},{i,51}]
Do[
uu[[i]]= u /. FindRoot[u-e Sin[u] == .04 Pi (i-26),{u,.04 Pi (i-26) }] ;
rr[[i,1]]=a (Cos[uu[[i]]]-e) ;
rr[[i,2]]=a Sqrt[1-e^2] Sin[uu[[i]]],
{i,51} ]

aJ= 5.203; eJ=.048; omegaJ=N[(15.5-100.4) Pi/180]; nodesJ=N[100.4 Pi/180]; inclinJ=N[1.31 Pi/180];
ParametricPlot3D[Evaluate[
    RotationMatrix3D[-omegaJ,-inclinJ,-nodesJ].{ aJ(Cos[u]-eJ), aJ Sqrt[1-eJ^2] Sin[u], 0 }
    ],{u,-Pi,Pi},
    AxesLabel->{x,y,z},ViewPoint->{2,-2,.4}]

Show[Graphics3D[Table[Point[RotationMatrix3D[-omega,-inclin,-nodes].rr[[i]]],{i,51}]]]

ParametricPlot3D[Evaluate[
    RotationMatrix3D[-omega,-inclin,-nodes].{ a(Cos[u]-e), a Sqrt[1-e^2] Sin[u], 0 }
    ],{u,-Pi,Pi},
    AxesLabel->{x,y,z},ViewPoint->{2,-2,.4}]

Show[%%%,%%,%,ViewPoint->{2,-2,.4}]
Show[%,PlotRange->{{-10,10},{-10,10},{-10,10}},ViewPoint->{0,0,2}]
Show[%%,PlotRange->{{-10,10},{-10,10},{-10,10}},ViewPoint->{2,2,.1}]
```

2 Homework

Turn in a printout showing each step as *Mathematica* solves the problem.

The orbital elements of Halley's comet during its 1986 trip through the inner Solar System are given above. The orbital elements of the Earth in 1986 are:

```
aE= 1; eE=.0167; omegaE=N[(102.8) Pi/180]; nodesE=0; inclinE=0;
```

On the date of Halley's closest approach to the Sun (1986 February 9), the Earth had $\omega t \approx 35^\circ$ Find the location of the Earth and Halley's comet for the 5 weeks before closest approach and 5 weeks after closest approach. Plot these locations along with the orbits.

The science fiction book *Heart of the Comet* by Gregory Benford and David Brin describe a trip to Halley's Comet when it returns in 2061. Gravitational perturbations due (mostly) to Solar System planets are predicted to slightly change Halley's orbit during its 2061 trip through the inner Solar System:

```
a= 17.74; e=.9666; omega=N[112.1 Pi/180]; nodes=N[59.4 Pi/180]; inclin=N[162.0 Pi/180];
aE= 1; eE=.0167; omegaE=N[(102.1) Pi/180]; nodesE=0; inclinE=0;
aV= .7233; eV=.0068; omegaV=N[(55.2) Pi/180]; nodesV=N[(76.5) Pi/180]; inclinV=N[(3.4) Pi/180];
```

On the date of Halley's closest approach to the Sun (2061 July 28), the Earth will have $\omega t \approx 203^\circ$ Find the location of the Earth and Halley's comet for the 5 weeks before closest approach and 5 weeks after closest approach. Do the same Venus (which has a very close approach with the comet); Venus has $\omega t \approx 81^\circ$ at time of closest approach. Plot these locations along with the orbits.

Extra Credit: When will Halley's comet be closest to Earth? Venus?

More Extra Credit: Our plots show "God's View" of the Solar System. We on Earth must view from Earth, and generally cannot with our eyes determine the distance to objects. Using your 11 positions of Halley's comet and Earth, find the direction vector from Earth to the comet. With Kirkman's help, plot these position vectors on a map of the stars. Newton, of course, had to do the reverse (and *much harder*) task of determining the comet orbit from the apparent location of the comet among the stars.