T6B.1:

$$\begin{aligned}\frac{1}{T} &= \frac{\partial S}{\partial U} \approx \frac{\Delta S}{\Delta U}\\ T &\approx \frac{\Delta U}{\Delta S} = \frac{35\text{ J}}{0.1\text{ J/K}} = 350\text{ K}\end{aligned}$$

T6B.5: According to Eq. T6.32 the energy levels of the orbitting electron in the H-atom are given by:

$$E_n = \frac{-13.6\text{ eV}}{n^2}$$

where $n = 1$ is the ground state and $n = 2$ is the first excited state. So $\Delta E = -13.6/4 + 13.6 = 10.2$ eV.

$$\frac{\Pr(E_2)}{\Pr(E_1)} = e^{-\Delta E/kT} = \exp\left(\frac{-10.2 \cdot 1.6022\times10^{-19}}{1.3807\times10^{-23}\cdot 9500}\right) = 3.88\times10^{-6}$$

where I've used the conversion factor: 1.6022×10^{-19} J/eV. As discussed on p. 107 there are actually four $n = 2$ states $(2s, 2p_x, 2p_y, 2p_z)$, each one of which would have the above probability, so the total probability of all $n = 2$ states is $4\times3.88\times10^{-6} = 1.55\times10^{-5}$ FYI: this fraction is small but quite measureable—about 3000× the similar fraction in the Sun.

T6S.4: Note: $S = k\ln(\Omega) = k(N\ln V + \frac{3}{2}N\ln U + \text{constant})$

$$\begin{aligned}\frac{1}{T} &= \frac{\partial S}{\partial U} = k\,\frac{\partial \ln\Omega}{\partial U} = \frac{3}{2}Nk\frac{\partial \ln U}{\partial U} = \frac{\frac{3}{2}Nk}{U}\\ U &= \frac{3}{2}NkT\end{aligned}$$

T6S.7: $E_n = \hbar\omega\left(n+\frac{1}{2}\right)$, for $n = 0, 1, 2\ldots$.

$$Z = \sum_{n=0}^{\infty}\exp(-E_n/kT) = \exp(-\hbar\omega/2kT)\sum_{n=0}^{\infty}\exp(-\hbar\omega n/kT)$$

This series can actually be exactly summed as it is geometric: define $r = \exp(-\hbar\omega/kT)$, then:

$$Z = \sqrt{r}\sum_{n=0}^{\infty}r^n = \frac{\sqrt{r}}{1-r} = \frac{1}{1/\sqrt{r}-\sqrt{r}} = \frac{1}{2}\,\frac{2}{\exp(+\hbar\omega/2kT)-\exp(-\hbar\omega/2kT)} = \frac{1}{2}\,\frac{1}{\sinh(\hbar\omega/2kT)}$$

I'll take "room temperature" to be 20°C = 293 K, in which case:

$$\hbar\omega/kT = \frac{1.0546\times10^{-34}\cdot3\times10^{14}}{1.3807\times10^{-23}\cdot293} = 7.8207$$

$$\begin{aligned}\Pr(E_n) &= \frac{e^{-E_n/kT}}{Z} = e^{-E_n/kT}\cdot(2\sinh(\hbar\omega/2kT))\\ \Pr(E_0) &= e^{-\ .5\cdot7.8207}\cdot(2\sinh(7.8207/2)) = 0.9995986\\ \Pr(E_1) &= e^{-1.5\cdot7.8207}\cdot(2\sinh(7.8207/2)) = 0.0004012\\ \Pr(E_2) &= e^{-2.5\cdot7.8207}\cdot(2\sinh(7.8207/2)) = 1.61\times10^{-7}\end{aligned}$$

Note: approximation of Z by adding three terms:

$$\begin{aligned}Z &= \frac{1}{2\sinh(\hbar\omega/2kT)} = \frac{1}{2\sinh(7.8207/2)} = 2.00415\times10^{-2}\\ &= \sqrt{r}\sum_{n=0}^{\infty}r^n \qquad \text{where: } r = \exp(-\hbar\omega/kT) = \exp(-7.8207) = .00040134\\ &\approx 2.00335\times10^{-2}\left(1+.00040134+1.61\times10^{-7}\right) = 2.00335\times10^{-2}\,(1.00040150) = 2.00415\times10^{-2}\end{aligned}$$

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Boltzmann_Factor.problems.txt
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A. The Boltzmann factor is:

$$\frac{N_1}{N_0} = e^{-\Delta E/kT}$$

For HCl:

$$\frac{\Delta E}{kT} = \frac{.37 \cdot 1.6022 \times 10^{-19}}{1.3807 \times 10^{-23} \cdot 298} = 14.4$$

$$\frac{N_1}{N_0} = e^{-14.4} = 5.53 \times 10^{-7}$$

For I_2:

$$\frac{\Delta E}{kT} = \frac{.027 \cdot 1.6022 \times 10^{-19}}{1.3807 \times 10^{-23} \cdot 298} = 1.05$$

$$\frac{N_1}{N_0} = e^{-1.05} = 0.349$$

Only I_2 is vibrationally excited so (at this temperature) only it would have additions to f for vibrations.