Summary of Lower Hybrid Wave Analysis at 1998-08-26 IP Shock

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1 Cold Plasma Definitions

If we consider the case of a cold uniform plasma with only linear waves, then we have from *Stix* [1962]:

$$S = 1 - \sum_s \frac{{\omega_{ps}}^2}{\omega^2 - {\Omega_{cs}}^2} \tag{1a}$$

$$D = \sum_s \frac{\Omega_{cs} {\omega_{ps}}^2}{\omega \left(\omega^2 - {\Omega_{cs}}^2\right)} \tag{1b}$$

$$P = 1 - \sum_s \frac{{\omega_{ps}}^2}{\omega^2} \tag{1c}$$

$$R = 1 - \sum_s \frac{{\omega_{ps}}^2}{\omega \left(\omega + \Omega_{cs}\right)} \tag{1d}$$

$$L = 1 - \sum_s \frac{{\omega_{ps}}^2}{\omega \left(\omega - \Omega_{cs}\right)} \tag{1e}$$

where Ω_{cs} is the gyrofrequency of species s, ω_{ps} is the plasma frequency of species s, and ω is the wave frequency. The dispersion relation, D(**k**, ω), can be simplified down if we assume the index of refraction, **n**, is parallel to the wave vector, **k**. Then we have:

$$D\left(\mathbf{k}, \omega\right) = An^4 - Bn^2 + RLP = 0 \tag{2}$$

where the terms A and B are defined by:

$$A = S\sin^2\theta + P\cos^2\theta \tag{3a}$$

$$B = RL\sin^2\theta + PS\left(1 + \cos^2\theta\right) \tag{3b}$$

which has the unique solutions of:

$$n^2 = \frac{B \pm F}{2A} \tag{4}$$

where F is defined by:

$$F = \left(RL - PS\right)^2 \sin^2\theta + 4P^2 D^2 \cos^2\theta \tag{5}$$

where we can see that F is always real. Since the terms A and B are real, then we can say that n^2 must either be purely real ($n^2 > 0$) or purely imaginary ($n^2 < 0$). If $n^2 < 0$, then the wave becomes evanescent (*i.e.* it damps out).

2 Turning Points

If we consider the case of an inhomogeneous plasma with dispersion relations of the form $b\kappa^2 + c = 0$, where κ is the propagation constant, b and c are functions of density, magnetic field strength, and position [*Stix*, 1962]. If the variation of plasma parameters is sufficiently slow, then we can argue that $dB_o/dx(dN_i/dx) \ll k_x B_o(k_x N_i)$. If we also assume that the scalar factor in the scalar wave equation (corresponds to the homogeneous plasma relation given by $b\kappa^2 + c = 0$), then we can say:

$$\frac{d^2E}{dx^2} + \kappa^2 E = 0 \tag{6}$$

where κ^2 = -c/b and now $\kappa = \kappa$(x). When we have the following two conditions:

$$|\frac{d^2\kappa}{dx^2}| \ll |\kappa \frac{d\kappa}{dx}| \tag{7a}$$

$$|\frac{d\kappa}{dx}| \ll |\kappa^2| \tag{7b}$$

we can find an approximate solution for E to Equation 6 using the WKB approximation of the form:

$$E \approx \frac{C_o}{\sqrt{\kappa}} e^{\pm i \int dx\, \kappa} \tag{8}$$

where C_o is some constant. In regions where b(x) or c(x) = 0, then the variation in κ^2 is rapid and Equation 8 is not a valid solution to Equation 6. To deal with this issue, we can approximate Equation 6 in the vicinity of c(x) = 0[b(x) = 0] using the linear[singular] turning point equation given by:

$$\frac{d^2E}{dx^2} + (x - x_o + i\varepsilon)\, \nu E = 0 \tag{9a}$$

$$\frac{d^2E}{dx^2} + \frac{\mu E}{(x - x_o + i\varepsilon)} = 0 \tag{9b}$$

where ν, μ, and ε are positive real constants and Equation 9a(Equation 9b) represents the linear(singular) turning point equation. The solution to Equation 9a is given by E = $E_+ + E_-$ and Equation 9b is given by E = $E_1 + E_2$, where the E_j's are given by:

$$E_\pm = A_\pm (x - x_o + i\varepsilon)^{1/2} J_{\pm 1/3}(\zeta_\pm) \tag{10a}$$

$$E_1 = B_1 (x - x_o + i\varepsilon)^{1/2} J_1(\zeta_1) \tag{10b}$$

$$E_2 = B_2 (x - x_o + i\varepsilon)^{1/2} Y_1(\zeta_1) \tag{10c}$$

where $A_\pm$ and $B_{1,2}$ are constants, J_n and Y_n are Bessel functions of the first and second kind, respectively, and I_n and K_n are the modified Bessel functions of the first and second kind given by:

$$J_n(x) = \sum_{j=0}^{\infty} \frac{(-1)^j}{j(n+j)} \left(\frac{x}{2}\right)^{2j+n} \tag{11a}$$

$$Y_n(x) = \frac{J_n(x)\cos(n\pi) - J_{-n}(x)}{\sin(n\pi)} \tag{11b}$$

$$I_n(x) = i^{-n} J_n(ix) \tag{11c}$$

$$= e^{-in\pi/2} J_n\left(x e^{i\pi/2}\right) \tag{11d}$$

$$K_n(x) = \frac{\pi}{2} \frac{I_{-n}(x) - I_n(x)}{\sin(n\pi)} \tag{11e}$$

and the terms ζ_j are given by:

$$\zeta_\pm = \frac{2}{3} \nu^{1/2} (x - x_o + i\varepsilon)^{3/2} \tag{12a}$$

$$\zeta_1 = 2\mu^{1/2} (x - x_o + i\varepsilon)^{1/2} \quad . \tag{12b}$$

The above solutions join smoothly to the following solutions at the turning point. For simplicity, let us define the following:

$$\beta_t \equiv (x - x_o + i\varepsilon)^{1/2} \tag{13}$$

which changes Equations 12a and 12b to:

$$\zeta_\pm = \frac{2}{3} \nu^{1/2} {\beta_t}^3 \tag{14a}$$

$$\zeta_1 = 2\mu^{1/2} \beta_t \quad . \tag{14b}$$

The solutions for Equations 9a and 9b at the turning point are:

$$E_{\pm} = (-1) \pm A_{\pm}\beta_t I_{\pm 1/3}(\zeta_{\pm}) \tag{15a}$$
$$E_1 = -B_1\beta_t I_1(\zeta_1) \tag{15b}$$
$$E_2 = -B_2\beta_t \{frac2\pi K_1(\zeta_1) \pm iI_1(\zeta_1)\} \tag{15c}$$

where the sign in Equation 15c is chosen based upon the sign of ε (*i.e.* $+$ for $\varepsilon > 0$).

If we consider a more complex wave equation of the form:

$$a\kappa_x{}^4 + (\Re[b] + i\Im[b])\,\kappa_x{}^2 + c = 0 \tag{16}$$

where a, b, and c are constants. Reflection and/or absorption occur at the critical layer if the following is satisfied:

$$\Im[b] \gg 4|ac| \quad \Rightarrow \text{Absorption} \tag{17a}$$
$$\Im[b] \ll 4|ac| \quad \Rightarrow \text{Reflection} \tag{17b}$$

which in practical application, Equation 17a looks like the following:

$$\left(\frac{\omega_{lh}\eta_{\perp}}{4\pi}\right)^2 \gg 6\frac{\beta_{\perp}}{\gamma^2}\left(1 + \frac{\omega_{lh}{}^4}{4\Omega_{ce}{}^2\Omega_{ci}{}^2}\right)\left(\frac{n_z{}^2 B_o{}^2}{4\pi N_e m_e c^2} - 1\right) \tag{18}$$

where we have assumed $\mathrm{T}_{\perp,e} = \mathrm{T}_{\perp,i}$ and $\beta_{\perp}$ is the perpendicular plasma beta and γ^2 is defined by:

$$\gamma^2 = \frac{4\pi(N_i M_i + N_e m_e)\,c^2}{B_o{}^2} \tag{19}$$

and where ω_{lh} is given by:

$$\frac{1}{\omega_{lh}{}^2} = \frac{1}{\Omega_{ci}{}^2 + \omega_{pi}{}^2} + \frac{1}{\Omega_{ci}\Omega_{ce}} \tag{20}$$

3 Lower Hybrid Wave Definitions

In general, when deriving the dispersion relation for lower hybrid waves (LHWs), one assumes that $\Omega_{ci} \ll \omega \ll \Omega_{ce} \ll \omega_{pe}$ and that $\cos^2\theta_{kB} \lesssim \mathrm{m}_e/\mathrm{M}_i$. Thus, one finds that $\mathrm{k}_{\|}/\mathrm{k}_{\perp} \lesssim \mathrm{m}_e/\mathrm{M}_i \ll 1$. We also know that LHWs can resonantly interact with unmagnetized ions ($\mathbf{k} \cdot \mathbf{V}_i$) and magnetized electrons ($\mathrm{k}_{\|}\ \mathrm{V}_{e,\|}$) at the same frequency ω [*Verdon et al.*, 2009a]. From this, we can see that LHWs can transfer perpendicular energy from the ions to parallel energy for the electrons, or vice versa. In either case, the result can be directed (acceleration) or random (heating) energization.

In the cold plasma limit, the ES dispersion relation for LHWs is given by:

$$\left(\frac{\omega}{\omega_{lh}}\right)^2 = 1 + \frac{m_e}{M_i}\cos^2\theta_{kB} \tag{21}$$

where ω_{lh} is defined by:

$$\omega_{lh}{}^2 \approx \frac{1}{1/\omega_{pi}{}^2 + 1/(\Omega_{ce}\Omega_{ci})} \tag{22a}$$
$$= \frac{(\Omega_{ce}\Omega_{ci})\omega_{pi}{}^2}{(\Omega_{ce}\Omega_{ci}) + \omega_{pi}{}^2} \tag{22b}$$

and we know the following:

$$\Omega_{ce}\Omega_{ci} = \Omega_{ce}{}^2\left(\frac{\omega_{pi}}{\omega_{pe}}\right)^2 \tag{23}$$

which leads to the final cold plasma ES approximation of:

$$\omega_{lh}{}^2 \approx \frac{(\Omega_{ce}\Omega_{ci})}{1 + (\Omega_{ce}/\omega_{pe})^2}\ . \tag{24}$$

When $\omega \sim \omega_{lh}$, the ions are unmagnetized and free to move $\perp$-$\mathbf{B}_o$ while electrons must move $\|$-$\mathbf{B}_o$. If $\delta\mathbf{E}$ is $\sim \perp$-$\mathbf{B}_o$, then the electron response time is greatly increased and LH-resonance can only occur when the electron response time is less than or comparable to the ion response time, or $\cos^2\theta_{kB} \lesssim m_e/M_i$. Notice that from Equation 21, the cold plasma ES LHW does not have a group velocity. However, when warm plasma effects or EM effects are added, the mode can propagate.

In the cold plasma limit, the EM dispersion relation for LHWs is given by:

$$\left(\frac{\omega}{\omega_{lh}}\right)^2 = \frac{1}{1+\omega_{pe}{}^2/k^2c^2}\left[1+\frac{\cos^2\theta_{kB}}{1+\omega_{pe}{}^2/k^2c^2}\right] \tag{25}$$

where this equation makes no assumption about the magnitude $\omega_{pe}{}^2/k^2c^2$.

Bingham et al. [2002] showed that an initial ion ring distribution given by:

$$f_{lh}\left(V_\|,V_\perp\right) = \frac{n_{ci}}{(2\pi)^{3/2}V_{Tci}{}^3}e^{-\frac{1}{2V_{Tci}{}^2}\left(V_\|{}^2+V_\perp{}^2\right)} + \frac{n_{ir}}{(2\pi)^2 V_{ir}V_{Tci}{}^2}e^{-\frac{1}{2V_{Tci}{}^2}\left[V_\|{}^2+(V_\perp{}^2-V_{ir}{}^2)\right]} \tag{26}$$

where V_{Tci} is the ion core thermal speed, V_{ir} is the ion ring speed, and $n_{ci}(n_{ir})$ is the ion core(ring) number density, can excite waves in the LH frequency range. The frequency resulting from this distribution is given by:

$$\omega = \omega_{lh}\left[1+\frac{1}{2}k^2\eta^2+\frac{m_e}{2M_i}\left(\frac{k_\|}{k_\perp}\right)^2-\left(\frac{\omega_{pe}}{\sqrt{2}kc}\right)^2\left(\frac{\omega_{pe}{}^2}{\omega_{pe}{}^2+\Omega_{ce}{}^2}\right)\right] \tag{27}$$

where ω_{lh} is given by:

$$\omega_{lh}{}^2 = \frac{(\Omega_{ce}\Omega_{ci})^2+(\omega_{pi}\Omega_{ce})^2}{\omega_{pe}{}^2+\Omega_{ce}{}^2} \tag{28a}$$

$$= \left(\frac{\omega_{pi}\Omega_{ce}}{\omega_{pe}}\right)^2\left[\frac{\omega_{pe}{}^2}{\Omega_{ce}{}^2}\frac{1+\left(\omega_{pi}\Omega_{ce}/\omega_{pe}{}^2\right)^2}{1+(\omega_{pe}/\Omega_{ce})^2}\right] \tag{28b}$$

$$= \omega_{pi}{}^2\left[\frac{1+\left(\omega_{pi}\Omega_{ce}/\omega_{pe}{}^2\right)^2}{1+(\omega_{pe}/\Omega_{ce})^2}\right] \tag{28c}$$

$$\approx \left(\frac{\omega_{pi}}{1+(\omega_{pe}/\Omega_{ce})^2}\right)^2 \tag{28d}$$

where the Equation 28d came from the approximation that $\omega_{pi}{}^2\Omega_{ce}{}^2/\omega_{pe}{}^4 \ll 1$. The η-term in Equation 27 is given by:

$$\eta = \left[\frac{3T_i}{\omega_{lh}{}^2M_i}+\left(\frac{2T_e}{\Omega_{ce}{}^2m_e}\right)\frac{\omega_{pe}{}^2}{\omega_{pe}{}^2+\Omega_{pe}{}^2}\right]^{1/2} \tag{29}$$

where $T_e(T_i)$ is the electron(ion) temperature. Thus, the 1/2 k^2 η^2 term in Equation 27 is the thermal correction and the last term is the EM correction. Resonance occurs at $\omega = \mathbf{k}\cdot\mathbf{V}_{ir}$ and the free energy associated with the ion ring feeds energy into the electrons. This is accomplished when the LHWs get concentrated into localized cavity structures by the modulational instability. The result is that the perpendicular ion energy gets transferred to the parallel electron energy.

4 Lower Hybrid Wave Literature

4.1 Marsch and Chang, [1983] and [1982]

Marsch and Chang [1983] and *Marsch and Chang* [1982] examined EMLHWs in the solar wind. They found the waves to have frequencies of $f_{ci} \ll f \ll f_{ce}$, they dissipate their wave energy through Landau interaction with the ions producing perpendicular ion heating, they propagate very obliquely to the field within a cone defined by $k_\|/k_\perp \leq 1/5$ and $k_\|/k_\perp \geq V_{Ti,\perp}/V_{Te,\|}$, and are thought to be driven unstable by the solar wind electron heat flux.

4.2 Zhang and Matsumoto, [1998]

Zhang and Matsumoto [1998] examined magnetic noise bursts (MNBs), using Geotail and Imp 8 spacecraft, near an IP shock on February 21, 1994. The plasma wave instruments (PWIs) onboard Geotail provide both waveform and dynamic spectral data. The waveform data is sampled at $\sim$12 kHz($\sim$0.083 ms resolution) for three B-field and two E-field components. The sweep frequency analyzer (SFA) is used to get the local plasma frequency. The IP shock arrives at Imp 8 at roughly 08:57 UT and at Geotail at roughly 09:03 UT.

Upstream of the IP shock, the MNBs are primarily created by waves with f $<$ 50 Hz and $\theta_{kB} \sim 9^\circ(171^\circ)$. Using the electric field data, the waves are found to have $\theta_{kB} > 90^\circ$, thus they propagate anti-parallel to the magnetic field. The waves are RH-polarized with respect to $\mathbf{B}_o$ but LH-polarized with respect to $\mathbf{k}$, thus they are whistler mode waves. They also compare the phase speed (cold plasma dispersion, $V_{whistler}$) to the $\mathbf{E} \times \mathbf{B}$ speed ($V_{E/B}$) finding that $V_{whistler} \sim V_{E/B} > V_{sw}$. The phase speed exceeding the solar wind speed is important to confirm that Doppler effects are not reversing the polarization.

Downstream of the shock, there are two types of MNBs which they call: Type **A** and Type **B**. Type **A** MNBs have f $<$ 50 and are composed of two types of waves, a longitudinal and transverse component.

1. **Longitudinal** $\Rightarrow$ Whistlers
   (a) $f \sim f_{lh}$
   (b) $\theta_{kB} \sim 10^\circ$ - 60°
   (c) RH-polarized
   (d) $V_{whistler} \sim V_{E/B}$
2. **Transverse** $\Rightarrow$ LHWs
   (a) $f_{ci} \ll f \lesssim f_{lh}$
   (b) $\theta_{kB} \sim 85^\circ$ - 90°
   (c) RH-polarized
   (d) $V_{whistler} \ll V_{E/B}$

Type **B** have f $<$ 50 Hz and f $\gtrsim$ 100 Hz (well separated in frequency).

1. **Waves with f $<$ 50 Hz** $\Rightarrow$ LHWs
   (a) f $\sim$ 10-20 Hz
   (b) $\theta_{kB} \sim 85^\circ$ - 90°
   (c) Both RH and LH-polarized
   (d) $V_{whistler} \ll V_{E/B}$
2. **Waves with f $\gtrsim$ 100 Hz** $\Rightarrow$ Whistlers
   (a) f $\sim$ 80-200 Hz
   (b) $\theta_{kB} \lesssim 35^\circ$
   (c) RH-polarized
   (d) $V_{whistler} \sim V_{E/B}$

The wave amplitudes were $\sim$0.2-0.6 nT peak-to-peak for the whistler-like waves and $\sim$1.5 nT.

4.3 Bell and Ngo, [1990]

Bell and Ngo [1990] derived analytical expressions for the consequences of a single normal mode scattering due to a discontinuity in a cold uniform plasma. There are four possible modes. For an incident whistler wave, two of the excited modes are quasi-electrostatic (QES) LHWs with short λ.

Assume a whistler wave is incident on a discontinuity in density with $N_{i,2} \neq N_{i,1}$ and the index of refraction is given by $n(\theta_{inc})$ and we know the index of refraction parallel to a boundary in the YZ-plane is $n_{zi} = n(\theta_{inc}) \cos\theta_{inc}$, and by Snell's law n_z must be conserved across the boundary.

The line $n_z = n_{zi}$ cuts through the surface of $\mathbf{n}$ at four points. The wave normal angles associated with these points define four normal modes which are solutions to Maxwell's equations in each region. Thus, each of the four possible solutions represent a propagating whistler mode wave. Two of the solutions lie near the resonance cone where $n(\theta) \rightarrow \infty$, which represent the QES LHWs of relatively short wavelength. The LHWs have $\mathbf{k} \times \mathbf{B}_o \approx 0$ while the EM whistlers have

$\mathbf{k} \cdot \mathbf{B}_o \approx 0$.

If we assume an EM whistler mode is incident on a density irregularity (width $\perp$-$\mathbf{B}_o \ll$ wavelength of incident wave) of length ΔL lies along $\mathbf{B}_o$, we find that two QES LHWs are produced on either side of the density irregularity propagating at a small angle δ with respect to $\mathbf{B}_o$. Thus, the group velocities of the QES LHWs are given by $\mathbf{V}_{g,ES} = \mathrm{V}_{\parallel}\, \hat{b}_o + \mathrm{V}_{\perp}\, [(\hat{n} \times \hat{b}_o) \times \hat{n}]$, where $\hat{n}$ is the vector normal to the density irregularity, $\mathrm{V}_{\parallel} = \mathrm{V}_{g,ES} \cos\delta$, and $\mathrm{V}_{\perp} = \mathrm{V}_{g,ES} \sin\delta$. Note that the wave vectors, $\mathbf{k}_{ES}$, of the QES LHWs are nearly orthogonal to $\mathbf{V}_{g,ES}$, thus they have a significant component along $\hat{n}$. Of course, this is specific to the case where $\mathbf{B}_o \cdot \hat{n} \approx 0$.

They arrive at a general solution for the index of refraction along the x-direction (parallel to the normal here) and $\mathbf{B}_o$ at angle χ with respect to density irregularity given by:

$$\alpha_4 {n_x}^4 + \alpha_3 {n_x}^3 + \alpha_2 {n_x}^2 + \alpha_1 n_x + \alpha_0 = 0 \tag{30}$$

where the α_i terms are given by:

$$\alpha_4 = S\cos^2\chi + P\sin^2\chi \tag{31a}$$
$$\alpha_3 = (P-S)\, n_z \sin 2\chi \tag{31b}$$
$$\alpha_2 = (P+S)\, {n_z}^2 + \left[S\left(1+\cos^2\chi\right) + P\sin^2\chi\right] {n_y}^2 - RL\cos^2\chi - PS(1+\sin^2\chi) \tag{31c}$$
$$\alpha_1 = (n_z \sin 2\chi)\left[(P-S)\left({n_y}^2 + {n_z}^2\right) + RL - PS\right] \tag{31d}$$
$$\begin{aligned}\alpha_0 = {} & S\left({n_y}^2+{n_z}^2\right)\left({n_y}^2+{n_z}^2\sin^2\chi\right) + P{n_z}^2\left({n_y}^2+{n_z}^2\right)\cos^2\chi \\ & - PS\left[{n_y}^2 + {n_z}^2\left(1+\cos^2\chi\right)\right] + PRL - RL\left({n_y}^2 + {n_z}^2\sin^2\chi\right)\end{aligned} \tag{31e}$$

where S, D, P, R, and L are defined by Equations 1a through 1e. In the small χ limit, the roots of Equation 30 can be simplified down to:

$$ {n_x}^{ES} \approx \frac{-\alpha_3 \pm \sqrt{{\alpha_3}^2 - 4\alpha_4\alpha_2}}{2\alpha_4} \tag{32a}$$
$$ {n_x}^{WM} \approx \frac{-\alpha_1 \pm \sqrt{{\alpha_1}^2 - 4\alpha_2\alpha_0}}{2\alpha_2} \tag{32b}$$

where the superscript $ES(WM)$ refers to the QES LHWs(EM whistlers). We can see that whenever $|\mathrm{n}_x| \gg |\mathrm{n}_z|$, then $|(\hat{k} \times \mathbf{E}) \times \hat{k}| \ll |(\hat{k} \cdot \mathbf{E})|$.

4.4 Cairns and McMillan, [2005]

Cairns and McMillan [2005] examined LHWs driven by LHDI finding that they could cause perpendicular ion heating and parallel electron heating of the high energy tail because they have $\omega/\mathrm{k}_{\parallel} \gg \omega/\mathrm{k}_{\perp}$. The LHDI, which in the presence of strong plasma gradients, acts like a fluid instability excited through the coupling of a LHW and a drift wave [*Davidson and Gladd*, 1975; *Huba et al.*, 1978]. When the gradients are weak, the LHDI is a kinetic instability driven by a resonance between ions and a drift wave. When in the presence of a finite plasma β, the LHDI exists as an ES and electromagnetic mode [*Davidson and Gladd*, 1975; *Huba et al.*, 1978]. The growth rate of the LHDI peaks at $\mathrm{k}\rho_e \approx 1$, for a broad range of frequencies near f_{lh} [*Davidson and Gladd*, 1975; *Cairns and McMillan*, 2005]. The mode is strongly unstable when the magnetic field gradient scale lenght, L_B, is comparable to ρ_i. The LHDI produces strong anomalous resistivity due to the wave's electric fields, $\delta\mathbf{E}_{\perp}$, perpendicular to the ambient magnetic field, $\mathbf{B}_o$, which create ($\delta\mathbf{E}_{\perp} \times \mathbf{B}_o$)-drifts that transport particles across $\mathbf{B}_o$. Thus, the LHDI causes cross-field diffusion which is an increase in entropy, thus irreversible and important for energy dissipation [*Coroniti*, 1985].

4.5 Walker et al., [2008]

Walker et al. [2008] used Cluster electric field measurements with the phase differencing technique at the terrestrial bow shock to investigate lower hybrid waves. A phase difference of zero implies linear polarization while a phase difference of $\pm\pi/2$ implies circular polarization. Wavelet spectrograms show significant enhancement in power just above the lower hybrid resonance frequency, $\mathrm{f}_{lh} = (\mathrm{f}_{ce}\ \mathrm{f}_{ci})^{1/2}$. They define any wave with circular polarization as whistler mode waves. One shoule note, however, that in the limit of large $\mathrm{k}_{\perp}$, LHWs are on the same branch of the dispersion relation as whistler waves.

The amplitude of the lower-hybrid-like waves were $\sim$1-3 mV/m and the whistler-like modes were of similar magnitude.

4.6 Verdon et al., [2009a]

Verdon et al. [2009b] examined rederived the dispersion relation for LHWs when considering warm plasma effects, EM effects, and ω_{pe}/Ω_{ce} ¿ 1. They found that as T_i increases, the LHW dispersion breaks up into a series of ion Bernstein waves. When this occurs, there are no modes near exact harmonics of Ω_{ci}. This is in agreement with previous studies that perturbed the ES limit of the LHW dispersion by including ion magnetization effects and only ion thermal effects. *Feng et al.* [1992] found similar results for IAWs propagating at large θ_{kB}; the mode breaks up into a series of ion Bernstein modes at low k-values. The regions where numerical solutions for $\Re(\omega)$ become ion Bernstein modes is where $|\Im(\omega)/\Re(\omega)|$ ($\approx$0.005) is larger than regions where the LH mode is continuous, which is also where the validity of weak damping becomes questionable.

They compare a number of dispersion relations, including:

$$\omega^2 = {\omega_{lh}}^2 \left(1 + \frac{M_i}{m_e} \cos^2\theta_{kB}\right) \tag{33a}$$

$$\left(\frac{\omega}{\omega_{lh}}\right)^2 = 1 + \frac{M_i}{m_e}\cos^2\theta_{kB} + 3\left[\frac{T_i}{T_e} + \frac{1}{4}\right]\left(\frac{kV_{Te}}{\Omega_{ce}}\right)^2 \tag{33b}$$

$$\left(\frac{\omega}{\omega_{lh}}\right)^2 = \frac{1}{1 + {\omega_{pe}}^2/(kc)^2}\left[1 + \frac{M_i}{m_e}\left(\frac{\cos^2\theta_{kB}}{1 + {\omega_{pe}}^2/(kc)^2}\right)\right] \tag{33c}$$

$$\left(\frac{\omega}{\omega_{lh}}\right)^2 = 1 + \frac{M_i}{2m_e}\cos^2\theta_{kB} - \frac{{\omega_{pe}}^2}{2k^2c^2} + \left[\frac{3T_i}{2T_e} + 1\right]\left(\frac{kV_{Te}}{\Omega_{ce}}\right)^2 \tag{33d}$$

where Equation 33a refers to the LH dispersion relation in a cold uniform plasma with only ES oscillations, Equation 33b refers to the LH dispersion relation with warm plasma effects added but the oscillations are assumed to be longitudinal, Equation 33c refers to the LH dispersion relation for a cold plasma but EM effects are included, and Equation 33d refers to the LH dispersion relation includes both EM and warm plasma effects. Their new analytical expression is given by:

$$\omega^2 = \frac{{\omega_{lh}}^2}{1 + {\omega_{pe}}^2/(kc)^2}\left[1 + \frac{M_i}{m_e}\left(\frac{\cos^2\theta_{kB}}{1 + {\omega_{pe}}^2/(kc)^2}\right) + W\left(\frac{kV_{Te}}{\Omega_{ce}}\right)^2\right] \tag{34a}$$

$$\begin{aligned} W = {} & 3\frac{T_i}{T_e}\left(1 + \frac{{\omega_{pe}}^2}{k^2c^2}\right) + \frac{{\omega_{pe}}^2}{2k^2c^2} + \frac{9}{2} - \frac{15 + 21{\omega_{pe}}^2/(kc)^2}{4\left(1 + {\omega_{pe}}^2/(kc)^2\right)} - \\ & \left[3\frac{{\omega_{pe}}^2}{k^2c^2} + \frac{1 - 6{\omega_{pe}}^2/(kc)^2}{4\left(1 + {\omega_{pe}}^2/(kc)^2\right)}\right]\frac{M_i}{m_e}\cos^2\theta_{kB} + \\ & 3\left(1 + \frac{{\omega_{pe}}^2}{k^2c^2}\right)\left[\frac{M_i/m_e\cos^2\theta_{kB} + {\omega_{pe}}^2/(kc)^2 - T_i/T_e}{1 + {\omega_{pe}}^2/(kc)^2 M_i/m_e\cos^2\theta_{kB}}\right]\frac{M_i}{m_e}\cos^2\theta_{kB} \end{aligned} \tag{34b}$$

where in the limit of small ${\omega_{pe}}^2/k^2c^2$ and small $(M_i/m_e)\cos^2\theta_{kB}$, Equation 34a reduces to:

$$\left(\frac{\omega}{\omega_{lh}}\right)^2 = 1 + \frac{M_i}{2m_e}\cos^2\theta_{kB} - \frac{{\omega_{pe}}^2}{2k^2c^2} + \frac{3}{2}\left[\frac{T_i}{T_e} + 1\right]\left(\frac{kV_{Te}}{\Omega_{ce}}\right)^2 \tag{35}$$

which differs from Equation 33d only in the last term. In every equation, *Verdon et al.* [2009b] has assumed ω_{lh} of the form described by Equation 20 in the limit that ${\omega_{pi}}^2 \gg {\omega_{ci}}^2$.

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