d^8 the difference between $Ti = 3d^2$ ← 2 electrons

$Ni = 3d^6$ ← 2 holes

$$d \Rightarrow l = 2 \;;\; \vec{2+2} = 0^+, 1^-, 2^+, 3^-, 4^+$$

$$\vec{\tfrac{1}{2}+\tfrac{1}{2}} = 0^-, 1^+$$

^1S ^1G ^1D ^3F ^3P

or find options in handout

max S, max L, min J $\Rightarrow$ $^3F_2$ for Ti

max J $\Rightarrow$ $^3F_4$ for Ni

The energy of all these possible states (except ^1S in Ti) see file TiNi.dat

generally $^3F < {}^1D < {}^3P < {}^1G < {}^1S$

2

old exam #5 : $e^{5i(x_1+x_2)} \sin(3(x_1-x_2)) = \psi$

$\cos \qquad = \tilde{\psi}$

Long way: $\partial_1 \to i5\psi + 3\tilde{\psi}$

$\partial_1^2 \to (i5)^2\psi + i15\tilde{\psi} + i15\tilde{\psi} - 9\psi$

$\partial_2 \to i5\psi - 3\tilde{\psi}$

$\partial_2^2 \to (i5)^2\psi - i15\tilde{\psi} - i15\tilde{\psi} - 9\psi$

$\partial_1^2 + \partial_2^2 = -2(25+9)\psi$

$$-\frac{\hbar^2}{2m}(\partial_1^2+\partial_2^2) = \underbrace{\frac{\hbar^2}{m}34}_{E}\psi$$

Trick: use CM & relative coordinates $X = \frac{x_1+x_2}{2}$, $x = x_1 - x_2$

$$-\frac{\hbar^2}{2m}(\partial_1^2+\partial_2^2) = -\frac{\hbar^2}{2\cdot \underbrace{2m}_{\text{total}}}\partial_X^2 \quad -\frac{\hbar^2}{2\underbrace{m/2}_{\text{reduced}}}\partial_x^2$$

$\psi = e^{i10X}\sin 3x$

$\partial_X^2 = (i10)^2\psi \qquad \partial_x^2 = -9\psi$

$$\left(\frac{\hbar^2}{4m}\cdot 100 + \frac{\hbar^2}{m}9\right)\psi = \frac{\hbar^2 34}{m}\psi \quad \checkmark$$

The spacial state is antisymmetric under exchange so if paired with symmetric spin state ... AOK for a fermion

text 5.7: dist: $\psi_a(x_1)\psi_b(x_2)\psi_c(x_3)$

$$\text{fermi} = \frac{1}{\sqrt{6}}\begin{vmatrix} \psi_a(x_1) & \psi_b(x_1) & \psi_c(x_1) \\ \psi_a(x_2) & \psi_b(x_2) & \psi_c(x_2) \\ \psi_a(x_3) & \psi_b(x_3) & \psi_c(x_3) \end{vmatrix} = \psi_a(1)\psi_b(2)\psi_c(3) + \psi_b(1)\psi_c(2)\psi_a(3) + \psi_c(1)\psi_a(2)\psi_b(3) - \psi_a(1)\psi_c(2)\psi_b(3) - \psi_b(1)\psi_c(2)\psi_a(3) - \psi_c(1)\psi_b(2)\psi_a(3)$$

boson → change all $-$ to $+$

Class 26 — note: $|h\rangle$ & $|z\rangle$ is much like $\downarrow$ & $\uparrow$ except boson

a i) seek symmetric state of 2 humans: $|h\rangle|h\rangle = \downarrow\downarrow$

ii) symmetric state of human & zombie: $\frac{1}{\sqrt{2}}(|h\rangle|z\rangle + |z\rangle|h\rangle)$

$= \frac{1}{\sqrt{2}}(\downarrow\uparrow + \uparrow\downarrow)$

b) abbreviate EYB as E

$E|h\rangle = \frac{1}{\sqrt{2}}(|h\rangle + |z\rangle) = \frac{1}{\sqrt{2}}(\downarrow + \uparrow)$

$E|z\rangle = |z\rangle = \uparrow$

E_1 operates on first wavefunction & E_2 on second

$(E_1 + E_2)|h\rangle|h\rangle = (E_1|h\rangle)|h\rangle + |h\rangle(E_2|h\rangle)$

$= \frac{1}{\sqrt{2}}\left([|h\rangle + |z\rangle]|h\rangle + |h\rangle[|h\rangle + |z\rangle]\right)$

$= \frac{1}{\sqrt{2}}(2|h\rangle|h\rangle + |z\rangle|h\rangle + |h\rangle|z\rangle)$

orthogonal

orthogonal

$\langle h|\langle h|(E_1 + E_2)|h\rangle|h\rangle = \frac{2}{\sqrt{2}}\underbrace{\langle h|\langle h||h\rangle|h\rangle}_{1}$

$(E_1 + E_2)\frac{1}{\sqrt{2}}(|h\rangle|z\rangle + |z\rangle|h\rangle) = \frac{1}{\sqrt{2}}\{(E_1|h\rangle)|z\rangle + (E_1|z\rangle)|h\rangle + |h\rangle(E_2|z\rangle) + |z\rangle(E_2|h\rangle)\}$

$= \frac{1}{\sqrt{2}}\{\frac{1}{\sqrt{2}}(|h\rangle + |z\rangle)|z\rangle + |z\rangle|h\rangle + |h\rangle|z\rangle + |z\rangle(\frac{1}{\sqrt{2}}(|h\rangle + |z\rangle))\}$

$= \frac{1}{\sqrt{2}}\{\frac{2}{\sqrt{2}}|z\rangle|z\rangle + (\frac{1}{\sqrt{2}} + 1)|h\rangle|z\rangle + (1 + \frac{1}{\sqrt{2}})|z\rangle|h\rangle\}$

ortho

$\frac{1}{\sqrt{2}}(\langle h|\langle z| + \langle z|\langle h|)\frac{1}{\sqrt{2}}\{\quad\}$

$= \frac{1}{2}\{(\frac{1}{\sqrt{2}} + 1) + (1 + \frac{1}{\sqrt{2}})\} = (1 + \frac{1}{\sqrt{2}})$

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$$\psi = \underbrace{R_{32}}_{\substack{n=3\\ \ell=2}} \left(\sqrt{\tfrac{2}{3}}\, \underbrace{Y_2^1}_{m=1} \chi_+ + \sqrt{\tfrac{1}{3}}\, \underbrace{Y_2^2}_{m=2} \chi_- \right)$$

CG – $Y_2^1 \uparrow = \sqrt{\frac{4}{5}} \left|\frac{5}{2}\frac{3}{2}\right\rangle - \sqrt{\frac{1}{5}} \left|\frac{3}{2}\frac{3}{2}\right\rangle$

$Y_2^2 \downarrow = \sqrt{\frac{1}{5}} \left|\frac{5}{2}\frac{3}{2}\right\rangle + \sqrt{\frac{4}{5}} \left|\frac{3}{2}\frac{3}{2}\right\rangle$

H: $\frac{-13.6\,eV}{3^2}$

L: $\ell = 2 \Rightarrow \ell(\ell+1)\hbar^2 = 6\hbar^2$

L_z $\frac{2}{3}$ change $m=1$ $\quad$ $\frac{1}{3}$ chance $m=2$

S_z $\frac{2}{3}$ chance $\uparrow$ $\quad$ $\frac{1}{3}$ chance $\downarrow$

J: $j = 5/2$ & $3/2$ $\quad$ $j(j+1)\hbar^2 = \frac{5}{2}\cdot\frac{7}{2}\hbar^2$ &

$\frac{3}{2}\frac{5}{2}\hbar^2$

$$\sqrt{\tfrac{2}{3}} \left(\sqrt{\tfrac{4}{5}} \left|\tfrac{5}{2}\tfrac{3}{2}\right\rangle - \sqrt{\tfrac{1}{5}} \left|\tfrac{3}{2}\tfrac{3}{2}\right\rangle \right)$$
$$+ \sqrt{\tfrac{1}{3}} \left(\sqrt{\tfrac{1}{5}} \left|\tfrac{5}{2}\tfrac{3}{2}\right\rangle + \sqrt{\tfrac{4}{5}} \left|\tfrac{3}{2}\tfrac{3}{2}\right\rangle \right)$$

$$\Big(\underbrace{\sqrt{\tfrac{8}{15}} + \sqrt{\tfrac{1}{15}}}_{\text{square to get prob } j=\frac{5}{2}} \Big) \left|\tfrac{5}{2}\tfrac{3}{2}\right\rangle + \Big(\underbrace{\sqrt{\tfrac{4}{15}} - \sqrt{\tfrac{2}{15}}}_{\text{square to get prob } j=3/2} \Big) \left|\tfrac{3}{2}\tfrac{3}{2}\right\rangle$$

6 $\quad m=+1 \Rightarrow \frac{4}{5} = \text{Prob } j = 5/2 \qquad \frac{1}{5} = \text{prob } j = 3/2$

7 $\quad$ Prob $j = 5/2$: $\frac{4}{5}\cdot\frac{2}{3} + \frac{1}{5}\cdot\frac{1}{3} = \frac{3}{5}$

$P = 3/2 \quad \frac{1}{5}\,\frac{2}{3} + \frac{4}{5}\cdot\frac{1}{3} = \frac{2}{5}$