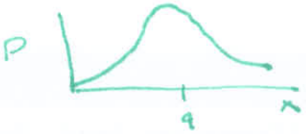
①

1.3 NB: $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$

$\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx = \frac{1}{2} \alpha^{-3/2} \sqrt{\pi}$



$$1 = A\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2} dx \underset{x-a=u}{=} A\int_{-\infty}^{\infty} e^{-\lambda u^2} du = A\sqrt{\frac{\pi}{\lambda}}$$

$\Rightarrow A = \sqrt{\frac{\lambda}{\pi}}$ check units: note $\lambda = \frac{1}{L^2}$ & $A = \frac{1}{L}$ is expected

$$\langle x\rangle = A\int_{-\infty}^{\infty} x e^{-\lambda(x-a)^2} dx \underset{\substack{x-a=u\\ x=u+a}}{=} A\int_{-\infty}^{\infty} \underbrace{(u+a) e^{-\lambda u^2}}_{0} du = a \underbrace{A\int_{-\infty}^{\infty} e^{-\lambda u^2} du}_{\substack{\shortparallel \\ 1}}$$

$= a$

$$\langle x^2\rangle = A\int_{-\infty}^{\infty} x^2 e^{-\lambda(x-a)^2} dx = A\int_{-\infty}^{\infty} (u+a)^2 e^{-\lambda u^2} du$$

$$= A\int_{-\infty}^{\infty} \left[\underbrace{u^2}_{\frac{1}{2\lambda}} + 2au + \underbrace{a^2}_{a^2} \right] e^{-\lambda u^2} du = \frac{1}{2\lambda} + a^2$$

$$\sigma_x^2 = \langle x^2\rangle - \langle x\rangle^2 = \frac{1}{2\lambda} + a^2 - a^2 = \frac{1}{2\lambda} \Rightarrow \sigma_x = \frac{1}{\sqrt{2\lambda}}$$

1.4 Recall $\rho = \Psi^* \Psi = \begin{cases} A^2\left(\frac{x}{a}\right)^2 \\ A^2\left(\frac{b-x}{b-a}\right)^2 \end{cases}$

$$1 = \int_{-\infty}^{\infty} \rho\, dx = A^2\left[\int_0^a \frac{x^2}{a^2} dx + \int_a^b \left(\frac{b-x}{b-a}\right)^2 dx \right]$$

$$= A^2\left[\frac{1}{3} a + \frac{1}{3}(b-a) \right] = A^2 \frac{b}{3} \Rightarrow A = \sqrt{\frac{3}{b}}$$

check units: expect $[\Psi] = \frac{1}{\sqrt{L}} = [A]$ correct as $[b] = L$

Prob $x \in [0,a] = \int_0^a \rho\, dx = A^2 \int_0^a \frac{x^2}{a^2} dx = A^2 \frac{a}{3} = \frac{a}{b}$

check $b = a \Rightarrow 1$ $b = 2a = 1/2$ makes sense

$$\langle x\rangle = \int_{-\infty}^{\infty} x\rho\, dx = A^2\left[\int_0^a \frac{x^3}{a^2} dx + \int_a^b \frac{x(b-x)^2}{(b-a)^2} dx \right]$$

$$= \frac{3}{b}\left[\frac{a^2}{4} + \frac{1}{(b-a)^2} \int_a^b \underbrace{b(b-x)^2 - (b-x)^3}_{\frac{1}{3}b(b-a)^3 - \frac{1}{4}(b-a)^4} \right]$$

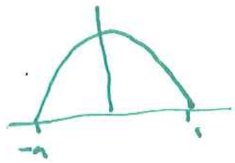
$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$-\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$

$$= \frac{3}{b}\left[\frac{a^2}{4} + \frac{1}{3} b(b-a) - \frac{1}{4}(b-a)^2 \right] = \frac{3}{b}\left[\frac{1}{12} b^2 + \frac{1}{6} ba \right]$$

$= \left[\frac{b+2a}{4} \right]$ check: $b = 2a \Rightarrow \langle x\rangle = a$ which makes sense

1.17



$$1 = \int_{-a}^{a} [A(a^2-x^2)]^2 dx = A^2 2 \int_0^a \underset{\hookrightarrow\, a^4 - 2a^2x^2 + x^4}{(a^2-x^2)^2} dx$$

$$= 2A^2 \left\{ a^4 \cdot a - 2a^2 \tfrac{1}{3} a^3 + \tfrac{1}{5} a^5 \right\} \qquad 1 - \tfrac{2}{3} + \tfrac{1}{5} = \tfrac{8}{15}$$

$$= \frac{16}{15} A^2 a^5 \qquad A = \sqrt{\frac{15}{16a^5}}$$

b) by symmetry $\langle x \rangle = 0$

c) If f is any real function : $\langle p \rangle = \langle f | p | f \rangle = 0$

Pf. $\int f \frac{\hbar}{i} \partial_x f \, dx = \frac{\hbar}{i} \int \frac{1}{2} \partial_x f^2 \, dx = \frac{\hbar}{2i} f^2 \Big|_{-\infty}^{\infty} = 0$

d) $$A^2 \int_{-a}^{a} x^2 (a^2-x^2)^2 dx = 2 \frac{15}{16a^5} \int_0^a x^2 (a^4 - 2a^2x^2 + x^4) dx$$

$$= \frac{15}{8a^5} \left\{ a^4 \tfrac{1}{3} a^3 - 2a^2 \tfrac{1}{5} a^5 + \tfrac{1}{7} a^7 \right\} \qquad \tfrac{1}{3} - \tfrac{2}{5} + \tfrac{1}{7} = \frac{8}{3 \cdot 5 \cdot 7}$$

$$= \frac{1}{7} a^2$$

e) $$A^2 \left(\frac{\hbar}{i}\right)^2 \int_{-a}^{a} (a^2-x^2) \overset{\substack{-2 \\ \uparrow \\ -2x \\ \uparrow}}{\partial_x^2 (a^2-x^2)} dx = A^2 \hbar^2 2 \int_{-a}^{a} (a^2-x^2) dx$$

$$= A^2 \hbar^2 4 \int_0^a (a^2-x^2) dx = A^2 \hbar^2 4 \left\{ a^2 \cdot a - \tfrac{1}{3} a^3 \right\} \qquad 1 - \tfrac{1}{3} = \tfrac{2}{3}$$

$$= \frac{15}{16a^2} \cdot \hbar^2 \cdot 4 \cdot \frac{2}{3} \cdot a^3 = \frac{5}{2} \frac{\hbar^2}{a^2}$$

f) $\langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{7} a^2 - 0^2$; $\sigma = \sqrt{\frac{a^2}{7}}$

g) $\langle p^2 \rangle - \langle p \rangle = \frac{5}{2} \frac{\hbar^2}{a^2}$ $\qquad \sigma = \frac{\hbar}{a} \sqrt{\frac{5}{2}}$

$$\sigma_x \sigma_p = \hbar \sqrt{\frac{5}{14}} \hookrightarrow .6 \checkmark$$

2-4 $\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$ ②

$$\langle x \rangle = \int_0^a \psi^* x \psi = \frac{2}{a} \int_0^a x \sin^2\Big(\overbrace{\frac{n\pi}{a} x}^{u}\Big)\, dx = \frac{2a}{(n\pi)^2} \int_0^{n\pi} u \sin^2 u\, du$$

$$= \frac{2a}{(n\pi)^2} \left[\frac{u^2}{4} - \underset{\downarrow\, 0}{\frac{u \sin 2u}{4}} - \underset{\downarrow\, 0}{\frac{\cos 2u}{8}} \right]_0^{n\pi} = \frac{2a}{(n\pi)^2} \frac{(n\pi)^2}{4} = \frac{a}{2} \checkmark$$

$$\langle x^2 \rangle = \int_0^a \psi^* x^2 \psi\, dx = \frac{2}{a} \int_0^a x^2 \sin^2\left(\frac{n\pi}{a} x\right) dx = \frac{2a^2}{(n\pi)^3} \int_0^{n\pi} u^2 \sin^2 u\, du$$

$$= \frac{2a^2}{(n\pi)^3} \left[\frac{u^3}{6} - \left(\frac{u^2}{4} - \frac{1}{8}\right) \underset{\downarrow\, 0}{\sin 2u} - \frac{x \cos 2u}{4} \right]_0^{n\pi}$$

$\cos 2\pi n = 1$

$$= \frac{2a^2}{(n\pi)^3} \left[\frac{(n\pi)^3}{6} - \frac{(n\pi)}{4} \right] = \frac{a^2}{3} \left[1 - \frac{3}{2} \frac{1}{(n\pi)^2} \right]$$

$$\langle p \rangle = \int_0^a \psi^* \frac{\hbar}{i} \partial_x \psi\, dx \underset{\psi}{=} \frac{\hbar}{i} \int_0^a \psi\, \partial_x \psi\, dx = \frac{\hbar}{i} \int_0^a \partial_x\left(\frac{\psi^2}{2}\right) dx$$

since ψ
is Real
here

Note: if ψ is real
$\langle p \rangle$ must be zero!

$$= \frac{\hbar}{i} \psi^2 \Big|_0^a = 0$$

$$\langle p^2 \rangle = -\hbar^2 \int_0^a \psi \psi'' dx = -\hbar^2 \frac{2}{a} \int_0^a \sin(kx) \left[-k^2 \sin(kx)\right] dx$$

$$= +\hbar^2 \frac{2}{a} k^2 \int_0^a \sin^2(kx)\, dx = \hbar^2 k^2 = \hbar^2 \left(\frac{n\pi}{a}\right)^2$$

$\int_0^a \sin^2(kx)\,dx \rightarrow \frac{1}{2} a$

$$\sigma_p^2 = \langle p^2 \rangle$$

$$\sigma_x^2 = \frac{a^2}{3} \left[1 - \frac{3}{2} \frac{1}{(n\pi)^2} \right] - \left(\frac{a}{2}\right)^2 = \frac{1}{12} a^2 - \frac{a^2}{2(n\pi)^2}$$

$$= \frac{a^2}{12} \left[1 - \frac{6}{\cancel{6}(n\pi)^2} \right]$$

$$\sigma_x^2 \sigma_p^2 = \frac{a^2}{12} \left[1 - \frac{6}{\cancel{4}(n\pi)^2} \right] \hbar^2 \left(\frac{n\pi}{a}\right)^2 = \frac{\hbar^2}{12} \left[(n\pi)^2 - 6 \right]$$

↑ clearly smallest
value is at
$n = 1$

for $n = 1$ $\sigma_x \sigma_p = .6\, \hbar$

2.5

$$1 = \int \Psi^* \Psi \, dx = A^2 \int (\psi_1 + \psi_2)^* (\psi_1 + \psi_2)\, dx = A^2 \int \underset{1}{\psi_1^* \psi_1} + \underset{0}{\psi_1^* \psi_2} + \underset{0}{\psi_2^* \psi_1} + \underset{1}{\psi_2^* \psi}$$

$$= A^2 \cdot 2 \Rightarrow A = \frac{1}{\sqrt{2}}$$

$$\left.\begin{aligned} E_1 &= \frac{\hbar^2 \pi^2}{2 m a^2} \\ E_2 &= 4 \cdot E_1 \end{aligned}\right\} \quad E_2 - E_1 = 3 E_1 = 3 \hbar \omega$$

$$\Psi = \frac{1}{\sqrt{2}} \left(\psi_1 e^{-i\frac{E_1 t}{\hbar}} + \psi_2 e^{-i\frac{E_2 t}{\hbar}} \right) \quad \text{Note } \psi_n \text{ are real here.}$$

$$\Psi^* \Psi = \frac{1}{2} \left(\psi_1^2 + \psi_2^2 + \psi_1 \psi_2 \left(\underbrace{e^{i\frac{(E_2 - E_1)t}{\hbar}} + e^{-i\frac{(E_2-E_1)t}{\hbar}}}_{2\cos\left[\frac{(E_2-E_1)t}{\hbar}\right] = 2\cos 3\omega t} \right) \right)$$

$$= \frac{1}{2} \left(\psi_1^2 + \psi_2^2 + 2\psi_1\psi_2 \cos(3\omega t) \right)$$

$$\langle x \rangle = \int_{\cancel{-\infty}\, 0}^{\cancel{\infty}} x \, \frac{1}{2} \left(\underset{\frac{a}{2}}{\psi_1^2} + \underset{\frac{a}{2}}{\psi_2^2} + 2\psi_1\psi_2 \cos(3\omega t) \right) dx$$

as $\langle x \rangle = \frac{a}{2}$ for stationary states

$$= \underbrace{\frac{2}{a} \int_{\cancel{-\infty}\, 0}^{a} x \sin\left(\frac{\pi}{a} x\right) \sin\left(\frac{2\pi}{a} x\right) dx}_{} \; \cos(3\omega t) + \frac{a}{2}$$

$$\hookrightarrow \frac{a^2}{\pi^2} \int_0^\pi u \sin u \sin(2u)\, du = \frac{a^2}{\pi^2} \int_0^\pi u \; 2 \sin^2 u \cos u \, du$$

$$= \frac{a^2}{\pi^2} \int_0^\pi u \, \partial_u (\sin^3 u) \frac{2}{3} \, du \underset{\text{parts}}{=} -\frac{a^2}{\pi^2} \int_0^\pi \sin^3 u \, \frac{2}{3} \, du$$

$$= -\frac{a^2}{\pi^2} \frac{2}{3} \Big[\underset{-\frac{2}{3}}{\frac{\cos^3 u}{3}} - \underset{2}{\cos u} \Big]_0^\pi = -\frac{a^2}{\pi^2} \frac{8}{9}$$

$$= \frac{a}{2} - \underset{\text{amplitude}}{\frac{16 a}{9\pi^2}} \cos(\underset{\text{angular freq}}{3\omega} t)$$

easy: by Eq 1.33 $\langle p \rangle = m \frac{d\langle x \rangle}{dt} = m 3\omega \frac{16a}{9\pi^2} \sin(3\omega t)$

Hard: $\langle p \rangle = A^2 \int \left[\underset{0}{\psi_1^* p \psi_1} + \psi_1^* p \psi_2 + \psi_2^* p \psi_1 + \underset{0}{\psi_2^* p \psi_2} \right] dx$

$$= A^2 \left\{ \int \underset{\hookrightarrow \text{parts}}{\sin(k_1 x)} \partial_x \sin(k_2 x) e^{\frac{i(E_1-E_2)t}{\hbar}} + \int \sin(k_2 x) \partial_x \sin(k_1 x) \, e^{\frac{i(E_2-E_1)t}{\hbar}} \right\} \frac{2}{a} \frac{\hbar}{i}$$

$$= A^2 \left\{ \int \sin(k_2 x) \partial_x \sin(k_1 x) \, dx \Big[\underbrace{e^{i\frac{E_2 - E_1}{\hbar}t} - e^{-i\frac{E_2-E_1}{\hbar}t}}_{2i \sin(3\omega t)} \Big] \right\} \frac{2}{a} \frac{\hbar}{i}$$

2-5 cont $\langle p \rangle = A^2 \frac{2}{a} \frac{\hbar}{i} 2i \sin(3\omega t) \int_0^a \sin\left(\frac{2\pi}{a}x\right) \frac{\pi}{a} \cos\left(\frac{\pi}{a}x\right) dx$

$\swarrow\ u = \frac{\pi}{a}x \qquad \sin(2\theta) = 2\sin\theta\cos\theta$

$= A^2 \frac{4}{a} \hbar \sin(3\omega t) \frac{4}{3}$

$= \frac{8}{3a}\hbar \sin(3\omega t)$

$\uparrow$

$\int_0^\pi 2\sin(u)\cos(u)\cos(u)\,du$

$\shortparallel$

$2\int_0^\pi \cos^2 u \sin u\, du \qquad \Chi = \cos u$

$\shortparallel \qquad d\Chi = -\sin u\, du$

$2\int_1^{-1} \Chi^2(-d\Chi) = \frac{4}{3}$

Note: $m\omega a = m \frac{\hbar\pi^2}{2ma^2} a$

$= \frac{\hbar}{a}\frac{\pi^2}{2}$

so same result as "easy"

$c_1 = \frac{1}{\sqrt{2}} \qquad c_2 = \frac{1}{\sqrt{2}}$ other $c = 0 \Rightarrow \langle H \rangle = c_1^2 E_1 + c_2^2 E_2$

equally likely: Prob = $\frac{1}{2}$ for E_1 & E_2; zero for all other E

2-7

$\frac{1}{2} = \int_0^{a/2} \psi^2 dx = A^2 \int_0^{a/2} x^2 dx = A^2 \frac{(a/2)^3}{3}$

$\frac{12}{a^3} = A^2$

$\Psi(x,t) = \sum c_n \psi_n e^{-iE_n t/\hbar}$

$\nwarrow \int \psi_n^* \psi_{start}\, dx$

Note: since the starting wave function is even (reflects thru $a/2$) it must be composed only from even functions, so $c_n = 0$ for n = even

Furthermore for these "even" functions:

$\int_0^a = 2\int_0^{a/2}$

$c_n = \sqrt{\frac{2}{a}}\, 2\int_0^{a/2} \sin\left(\frac{n\pi}{a}x\right) Ax\, dx$

$= \sqrt{\frac{2}{a}}\, 2A\left(\frac{a}{n\pi}\right)^2 \int_0^{n\pi/2} \sin u\; u\, du$

$\hookrightarrow \left[\sin u - u\cos u\right]_0^{n\pi/2} = 1 \text{ or } -1 \Leftarrow$ alternates

$= \sqrt{\frac{2}{a}}\, 2\sqrt{\frac{12}{a^3}} \left(\frac{a}{n\pi}\right)^2 (\pm 1)$

$= \frac{4\sqrt{6}}{\pi^2 n^2}(\pm 1) \qquad P_1 = c_1^2 = \frac{16\cdot 6}{\pi^4}$

Dwight $\nearrow$ $\frac{48\cdot 12}{\pi^2/8}$

$\langle E \rangle = \sum_{n\ odd} \frac{16\cdot 6}{\pi^4 n^4} \frac{\hbar^2\pi^2 n^2}{2ma^2} = \frac{48\hbar^2}{\pi^2 ma^2} \sum_{odd} \frac{1}{n^2} = \frac{6\hbar^2}{ma^2}$

or Sum[1/n^2, {n,1,Infinity,2}]

alt: $\langle H \rangle = \int \psi^* \frac{p^2}{2m}\psi\, dx \overset{\text{trick!}}{=} \frac{1}{2m}\int (p\psi)^*(p\psi)\, dx$

$= \frac{1}{2m}\, 2\int_0^{a/2} A^2 dx \left(\frac{\hbar}{i}\right)^*\left(\frac{\hbar}{i}\right) = \frac{\hbar^2}{m}A^2\frac{a}{2} = \frac{6\hbar^2}{ma^2}$ ✓

2-12 recall: $x = \sqrt{\frac{\hbar}{m\omega}}\,\frac{1}{\sqrt{2}}(a_+ + a_-)$ ⑦

$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$p = i\sqrt{\hbar m\omega}\,\frac{1}{\sqrt{2}}(a_+ - a_-)$

$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$

$\langle m|n\rangle = \delta_{mn}$

$\langle x\rangle = 0$ expected as symmetric potential

$\langle p\rangle = 0$ expected as ψ is real

$\Big\}$ confirm as $\langle n|\underbrace{a_\pm|n\rangle}_{\sim|n\pm1\rangle} = 0$

$$\langle x^2\rangle = \frac{\hbar}{m\omega}\frac{1}{2}\langle n|(a_+ + a_-)^2|n\rangle$$

$$\to \underset{0}{a_+^2} + a_+a_- + a_-a_+ + \underset{0}{a_-^2}$$

$$= \frac{\hbar}{m\omega}\frac{1}{2}\langle n|\,a_+a_- + a_-a_+\,|n\rangle$$

a_+: $\sqrt{n}\sqrt{n}\,|n\rangle = n\,|n\rangle$

a_-: $\sqrt{n}\,|n-1\rangle$

a_-: $\sqrt{n+1}\,\sqrt{n+1}\,|n\rangle = (n+1)\,|n\rangle$

a_+: $\sqrt{n+1}\,|n+1\rangle$

$$= \frac{\hbar}{m\omega}\frac{1}{2}(n + n+1) = \frac{\hbar}{m\omega}(n + 1/2)$$

$$\langle p^2\rangle = -(\hbar m\omega)\frac{1}{2}\langle n|(a_+ - a_-)^2|n\rangle$$

$$\to \underset{0}{a_+a_+} - a_+a_- - a_-a_+ + \overset{0}{a_-^2}$$

$$= +(\hbar m\omega)\frac{1}{2}\underbrace{\langle n|\,a_+a_- + a_-a_+\,|n\rangle}_{\text{as before } = 2n+1}$$

$$= (\hbar m\omega)(n + 1/2)$$

$$\langle T\rangle = \frac{1}{2m}\langle p^2\rangle = \frac{\hbar\omega}{2}(n + 1/2)$$

FYI: $\langle V\rangle = \frac{1}{2}k\langle x^2\rangle = \frac{1}{2}m\omega^2\langle x^2\rangle = \frac{\hbar\omega}{2}(n+1/2)$ also

$$\Delta x\,\Delta p = \sqrt{\frac{\hbar}{m\omega}}\sqrt{\hbar m\omega}\,(n+1/2) = \hbar(n+1/2)$$

2-13 (very similar to 2-5)

$$1 = \langle \psi | \psi \rangle = |A|^2 \langle 3\psi_0 + 4\psi_1 | 3\psi_0 + 4\psi_1 \rangle$$

$$= A^2 \left\{ 9 \underbrace{\langle \psi_0 | \psi_0 \rangle}_{1} + 12 \left[\underbrace{\langle \psi_0 | \psi_1 \rangle}_{0} + \underbrace{\langle \psi_1 | \psi_0 \rangle}_{0} \right] + 16 \underbrace{\langle \psi_1 | \psi_1 \rangle}_{1} \right\}$$

$$= A^2 \cdot 25 \Rightarrow A = 1/5$$

$$\psi = c_0 \psi_0 e^{-\frac{iE_0 t}{\hbar}} + c_1 \psi_1 e^{-\frac{iE_1 t}{\hbar}} \quad \text{where} \quad c_0 = A \cdot 3 = 3/5$$

$$c_1 = A \cdot 4 = 4/5$$

$$|\psi|^2 = |c_0|^2 |\psi_0|^2 + |c_1|^2 |\psi_1|^2 \qquad E_i = \hbar\omega(i + 1/2)$$

$$+ c_1^* c_0 \psi_1^* \psi_0 e^{i\omega t} + \psi_0^* \psi_1 e^{-i\omega t} c_0^* c_1 \qquad E_1 - E_0 = \hbar\omega$$

$$\langle x \rangle = |c_0|^2 \overset{0}{\langle 0|x|0\rangle} + |c_1|^2 \overset{0}{\langle 1|x|1\rangle}$$

$$+ c_1^* c_0 \underbrace{\langle 1|x|0\rangle}_{\sqrt{\frac{\hbar}{m\omega}} \frac{1}{\sqrt{2}}} e^{i\omega t} + c_0^* c_1 \underbrace{\langle 0|x|1\rangle}_{\sqrt{\frac{\hbar}{m\omega}} \frac{1}{\sqrt{2}}} e^{-i\omega t}$$

using $a_+|0\rangle = |1\rangle$ using $a_-|1\rangle = |0\rangle$

$$= \frac{12}{25} \sqrt{\frac{\hbar}{m\omega}} \sqrt{2} \underbrace{\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2} \right)}_{\cos(\omega t)}$$

$$\langle p \rangle = |c_0|^2 \langle 0|p|0\rangle + |c_1|^2 \langle 1|p|1\rangle$$

$$+ c_1^* c_0 \underbrace{\langle 1|p|0\rangle}_{i\sqrt{\hbar m\omega} \frac{1}{\sqrt{2}}} e^{i\omega t} + c_0^* c_1 \underbrace{\langle 0|p|1\rangle}_{-i\sqrt{\hbar m\omega} \frac{1}{\sqrt{2}}} e^{-i\omega t}$$

using $a_+|0\rangle = |1\rangle$ using $|a_-|1\rangle = |0\rangle$

$$= \frac{12}{25} \sqrt{\hbar m\omega} \sqrt{2} \underbrace{\left(i \frac{e^{i\omega t} - i e^{-i\omega t}}{2} \right)}_{-\sin(\omega t)}$$

indeed $\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m}$ $\quad \langle -\partial_x V \rangle = \langle -kx \rangle = -m\omega^2 \langle x \rangle = \frac{d\langle p\rangle}{dt}$ ✓

E_0 with Prob $= c_0^2 = \frac{9}{25}$ $\qquad E_1$ with prob $= c_1^2 = \frac{16}{25}$

For H_{10} $E' = 21$

$$a_2 = -\frac{21-1}{2} a_0 = -10 a_0 \checkmark$$

$$a_4 = -\frac{21-5}{4 \cdot 3} a_2 = \frac{-4}{3} a_2 \checkmark$$

$$a_6 = -\frac{21-9}{6 \cdot 5} a_4 = \frac{-2}{5} a_4 \checkmark$$

$$a_8 = -\frac{21-13}{8 \cdot 7} a_6 = \frac{-1}{7} a_6 \checkmark$$

$$a_{10} = -\frac{21-17}{10 \cdot 9} a_8 = \frac{-2}{45} a_8 \checkmark$$

see class3.m

2-19 Note time dependent part: $e^{-i\omega t}$ will cancel in $\psi^*\psi$

$$J = \frac{\hbar}{2mi}\psi^* \overset{\leftrightarrow}{\partial}\psi = \frac{\hbar}{2mi}\left[ik|A|^2 - -ik|A|^2\right] = \frac{\hbar k}{m}|A|^2 \quad \frac{\text{"}p\text{"}}{m} = \text{"}v\text{"} \checkmark$$

2-21 $$1 = \int_{-\infty}^{\infty} A^2 e^{-2q|x|}\,dx = 2\int_0^{\infty} A^2 e^{-2qx}\,dx = 2A^2\left[\frac{e^{-2qx}}{-2q}\right]_0^{\infty}$$

$$= \frac{A^2}{q} \Rightarrow A = \sqrt{q} \quad \text{check units: } \frac{1}{\sqrt{L}}$$

$$\phi(k) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-ikx} A e^{-q|x|}\,dx = \sqrt{\frac{q}{2\pi}}\left\{\int_{-\infty}^{0} e^{(q-ik)x}\,dx + \int_0^{\infty} e^{(-q-ik)x}\,dx\right.$$

$$= \sqrt{\frac{q}{2\pi}}\left\{\frac{1}{q-ik} + \frac{1}{q+ik}\right\} = \sqrt{\frac{q}{2\pi}}\left\{\frac{2q}{q^2+k^2}\right\}$$

$$\psi = \int_{-\infty}^{\infty} \phi(k)\frac{1}{\sqrt{2\pi}} e^{i\left(kx - \frac{\hbar k^2}{2m}t\right)}\,dk$$

$$\Delta x\,\Delta k \sim \frac{1}{q}\cdot q \sim 1$$

2-24. clearly $\delta(cx)$ is delta function like: zero except at $x=0$

but is $\int \delta(cx)dx = 1$? (No)

$$\int_{-\infty}^{\infty}\delta(cx)\,dx = \frac{1}{c}\int_{-\infty}^{\infty}\underset{u}{\delta(cx)}\,\underset{du}{c\,dx} = \frac{1}{c}\left\{\begin{matrix} c>0: \int_{-\infty}^{\infty}\delta u\,du = 1 \\ c<0: \int_{\infty}^{-\infty}\delta(u)du = -1\end{matrix}\right\} = \frac{1}{|c|}$$

So $\delta(cx)$ acts just like $\frac{1}{|c|}\delta(x)$

check units: $[\delta(cx)] = \frac{1}{cx}$ $\quad [\delta(x)] = \frac{1}{x}$ ✓

2-34 a) $$\left.\begin{matrix} e^{ikx} + Re^{-ikx} \\ E = \frac{\hbar^2k^2}{2m}\end{matrix}\right| \begin{matrix} Te^{-kx} \\ E = -\frac{\hbar^2k^2}{2m} + V_0\end{matrix}$$

$$\left.\begin{matrix}\psi\text{ cont: } & 1 + R = T \\ \psi'\text{ cont: } & ik(1-R) = -kT\end{matrix}\right\} 1+R = \frac{ik}{k}(R-1) \Rightarrow \frac{1+\frac{ik}{k}}{-1+\frac{ik}{k}} = R$$

$$|R|^2 = 1 \checkmark$$

Faster: since on rhs ψ = real, Flux = 0 hence incoming / outgoing flux must be same

21, 23

2.23

(a) Does interval $(-3, 1)$ include $x = -2$ … yes

result $= (x^3 - 3x^2 + 2x + 1)\Big|_{x=-2} = -23$

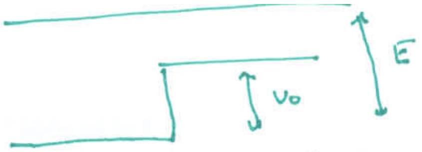
(b) Does interval $(0, \infty)$ include $x = \pi$ … yes

result $= (\cos(3x) + 2)\Big|_{x=\pi} = 2$

(c) Does interval $(-1, 1)$ include $x = 2$ NO

result $= 0$

2-34



⑧

$$\frac{\ell}{k} = \sqrt{\frac{E-V_0}{E}}$$

$1 + Re^{-ikx}$ $\qquad$ Te^{ilx}

$$E = \frac{\hbar^2 k^2}{2m} \qquad \frac{\hbar^2 \ell^2}{2m} + V_0 = E$$

$$\frac{\ell^2}{k^2} = \frac{E-V_0}{E} \leftarrow \frac{\frac{\hbar^2\ell^2}{2m} = E - V_0}{\frac{\hbar^2 k^2}{2m} = E}$$

$$\left.\begin{array}{l} \text{Flux: } \Rightarrow 1^2 \frac{\hbar k}{m} \qquad \Rightarrow |T|^2 \frac{\hbar \ell}{m} \\ \Leftarrow |R|^2 \frac{\hbar k}{m} \end{array}\right\} \text{transmission} = \frac{|T|^2 \frac{\hbar \ell}{m}}{|1|^2 \frac{\hbar k}{m}} = |T|^2 \boxed{\frac{\ell}{k}}$$

$$\left.\begin{array}{l} \psi \text{ cont: } \quad 1 + R = T \\ \psi' \text{ cont: } \quad ik(1-R) = i\ell T \end{array}\right\} \begin{array}{l} 1 + R = T \\ 1 - R = \frac{\ell}{k} T \\ \hline 2 = (1 + \frac{\ell}{k}) T \end{array}$$

$$\frac{2}{(1 + \ell/k)} = T$$

or $1 + R = \frac{k}{\ell}(1 - R)$

$$(1 - \frac{k}{\ell}) = -R(1 + \frac{k}{\ell}$$

$$\frac{\frac{k}{\ell} - 1}{\frac{k}{\ell} + 1} = R$$

$$\| $$

$$\frac{1 - \ell/k}{1 + \ell/k}$$

$$\left.\begin{array}{l} \%\text{ transmss.} = \frac{4}{(1+\ell/k)^2}\frac{\ell}{k} \\ \%\text{ reflecte} = |R|^2 = \frac{(1-\ell/k)^2}{(1+\ell/k)^2} \end{array}\right\} \frac{(1-\frac{\ell}{k})^2 + 4\frac{\ell}{k}}{(1+\ell/k)^2} = \frac{(1+\ell/k)^2}{(1+\ell/k)^2} = 1$$

A-8

$$A+B = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 3 \\ 3i & 3-2i & 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 & -i \\ 0 & 1 & 0 \\ i & 3 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 1+3i & 3i \\ 4+3i & 9 & 6-2i \\ 6i & 6-2i & 6 \end{pmatrix}$$

$$A^T = \tilde{A} = \begin{pmatrix} -1 & 2 & 2i \\ 1 & 0 & -2i \\ i & 3 & 2 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 0 & -i \\ 0 & 1 & 0 \\ i & 3 & 2 \end{pmatrix} \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 3 \\ 6+3i & -3i & 12 \end{pmatrix}$$

$$A^* = \begin{pmatrix} -1 & 1 & -i \\ 2 & 0 & 3 \\ -2i & +2i & 2 \end{pmatrix}$$

$$[A, B] = AB - BA \qquad \begin{pmatrix} -3 & 1+3i & 3i \\ 2+3i & 9 & 3-2i \\ -6+3i & 6+i & -6 \end{pmatrix}$$

$$A^+ = \begin{pmatrix} -1 & 2 & -2i \\ 1 & 0 & +2i \\ -i & 3 & 2 \end{pmatrix}$$

$$B^{-1} = \frac{1}{\det B} \begin{pmatrix} 2 & -3i & i \\ 0 & 3 & 0 \\ -i & -6 & 2 \end{pmatrix} \qquad -\begin{vmatrix} 0 & -i \\ 3 & 2 \end{vmatrix}$$

$\operatorname{Tr} B = 5$

Crammer's Rule: sub determinants from transpose with signs

$$-\begin{vmatrix} 0 & 0 \\ i & 2 \end{vmatrix}$$

$\det B = 4 + 0 + 0 - 1 = 3$

note: inverse 2x2: $\dfrac{\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}{ad - bc}$

A-9

$$Aa = \begin{pmatrix} -1 & 1 & i \\ 2 & 0 & 3 \\ 2i & -2i & 2 \end{pmatrix} \begin{pmatrix} i \\ 2i \\ 2 \end{pmatrix} = \begin{pmatrix} 3i \\ 6+2i \\ 6 \end{pmatrix}$$

$$a^{+}b = (-i, -2i, 2) \begin{pmatrix} 2 \\ 1-i \\ 0 \end{pmatrix} = -2i \; -2i \; -2 \; = -2-4i$$

$$a^{T} B b = (i \;\; 2i \;\; 2) \begin{pmatrix} 2 & 0 & -i \\ 0 & 1 & 0 \\ i & 3 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1-i \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 1-i \\ 3-i \end{pmatrix} = \underset{6-2i}{4i + 2i + 2} = 8+4i$$

$$a b^{+} = \begin{bmatrix} i \\ 2i \\ 2 \end{bmatrix} [2, 1+i, 0] = \begin{pmatrix} 2i & i-1 & 0 \\ 4i & 2i-2 & 0 \\ 2i & 2+2i & 0 \end{pmatrix}$$

A-26 $\det T = 8 -1 -1 -2 -2 -2 = 0$ ($\Rightarrow \exists$ a zero eigenvalue)

$\mathrm{Tr}\, T = 6$

$$\det \begin{bmatrix} 2-\lambda & i & 1 \\ -i & 2-\lambda & i \\ 1 & -i & 2-\lambda \end{bmatrix} = (2-\lambda)^3 -1 -1 -(2-\lambda) -(2-\lambda) -(2-\lambda)$$

$$= 8 - 12\lambda + 6\lambda^2 - \lambda^3 - 8 + 3\lambda$$

$$= 9\lambda + 6\lambda^2 - \lambda^3 = -\underset{\substack{\uparrow \\ 0}}{\lambda}\,(\lambda-3)^2 \quad \nwarrow \text{double root} : 3$$

seek vector for $\lambda = 0$

$$\begin{pmatrix} 2 & i & 1 \\ -i & 2 & i \\ 1 & -i & 2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0 \qquad \lambda = 0 \to \begin{pmatrix} -1 \\ -i \\ 1 \end{pmatrix} = \overrightarrow{W_0}$$

↑ this row is $\mathrm{row}_1 - i\,\mathrm{row}_2$

add $-2i \cdot \mathrm{row}_2$ to row_1

$$\begin{pmatrix} 0 & -3i & +3 \\ -i & 2 & i \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \to \text{so } v_2 = -i v_3 \qquad \text{say } v_3 = 1 \Rightarrow v_2 = -i$$

$$\mathrm{row}_2 \Rightarrow v_1 = -1$$

Note: the other eigenvectors must be orthogonal to this (in fact since the 2d space $\perp$ to $\overrightarrow{w_0}$ ~~must~~ any vector $\perp$ to $\overrightarrow{w_0}$ must be - I'll use $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ which checks

For my 3RD eigenvector, I select something $\perp$ to both

$$\vec{w}_0^{*} \times \vec{w}_1 \sim \begin{pmatrix} -1 \\ 2i \\ 1 \end{pmatrix}$$

A-25

a) $\tilde{T} = \begin{pmatrix} 1 & 1+i \\ 1-i & 0 \end{pmatrix}$; $\tilde{T}^{*} = \begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}$ ✓

b) $\begin{pmatrix} 1-\lambda & 1-i \\ 1+i & -\lambda \end{pmatrix} = \lambda(\lambda-1) - 2 = 0$

$\hookrightarrow (\lambda+1)(\lambda-2) < \begin{matrix} 2 \\ -1 \end{matrix}$ eigenvalues

eigen value = 2

$\begin{pmatrix} -1 & 1-i \\ 1+i & -2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0 \rightarrow a = (1-i)b$

take $b=1 \rightarrow \begin{pmatrix} 1-i \\ 1 \end{pmatrix}$

normalize: $\frac{1}{\sqrt{3}}\begin{pmatrix} 1-i \\ 1 \end{pmatrix}$

check

$\begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}\begin{pmatrix} 1-i \\ 1 \end{pmatrix} = \begin{pmatrix} 2(1-i) \\ 2 \end{pmatrix}$ ✓

eigen value = -1

$\begin{pmatrix} 2 & 1-i \\ 1+i & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0 \rightarrow b = -(1+i)a$

take $a=1 \longrightarrow \begin{pmatrix} 1 \\ -(1+i) \end{pmatrix}$

normalize: $\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -(1+i) \end{pmatrix}$

check

$\begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -(1+i) \end{pmatrix} = \begin{pmatrix} 1-2 \\ 1+i \end{pmatrix}$ ✓

orthogonal? $\begin{pmatrix} 1-i \\ 1 \end{pmatrix}^{\dagger} \cdot \begin{pmatrix} 1 \\ -(1+i) \end{pmatrix} = (1+i) - (1+i) = 0$

h

diagonalized form has eigenvalues on diagonal

$\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow \begin{matrix} \det = -2 \\ \mathrm{Tr} = 1 \end{matrix}$ ✓ $\longleftarrow \det\begin{pmatrix} 1 & 1+i \\ 1-i & 0 \end{pmatrix} = -2$ ✓

3-13

$$[AB,C] = ABC - CAB = \underbrace{ABC - ACB}_{A[B,C]} + \underbrace{ACB - CAB}_{[A,C]B}$$

$$[x^n, \tfrac{\hbar}{i}\partial_x] f = x^n \tfrac{\hbar}{i}\partial_x f - \tfrac{\hbar}{i} \underbrace{\partial_x (x^n f)} \rightarrow n x^{n-1} f + x^n f'$$

$$= -\tfrac{\hbar}{i} n x^{n-1} f = i\hbar n x^{n-1} f$$

argue any function works same

$$[V, \tfrac{\hbar}{i}\partial_x] f = V \tfrac{\hbar}{i}\partial_x f - \tfrac{\hbar}{i}\partial_x (V f)$$

$$= -\tfrac{\hbar}{i} (\partial_x V) f$$

* * *

method 1. $[p^2, x] = p \overbrace{[p,x]}^{\hbar/i} + [p,x] p$

$$= 2\tfrac{\hbar}{i} p$$

method 2. $[p^2, x] f = -\hbar^2 \{ \underbrace{\partial_x^2 (x f)}_{\underbrace{\partial_x (f + x f')}_{2f' + x f''}} - x \partial_x^2 f$

$$= -\hbar^2 2 \partial_x f$$

$$= 2\tfrac{\hbar}{i} p f$$

Induction: have proved case $n=2$; show: if n then $n+1$

$$[p^{n+1}, x] = p[p^n, x] + [p,x] p^n$$

$$= n\tfrac{\hbar}{i} p\, p^{n-1} + \tfrac{\hbar}{i} p^n = \tfrac{\hbar}{i} (n+1) p^n$$

3-37 $H = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}$ eigen values / vectors

$$\begin{array}{ccc} c & a+b & a-b \\ \begin{pmatrix}0\\1\\0\end{pmatrix} & \begin{pmatrix}1\\0\\1\end{pmatrix} & \begin{pmatrix}1\\0\\-1\end{pmatrix} \\ \uparrow & \downarrow & \downarrow \\ \psi_0 & \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix} & \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix} \\ & \uparrow & \uparrow \\ & \psi_+ & \psi_1 \end{array}$$

(a) $|S(t)\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix} e^{-ict/\hbar}$

(b) $\begin{pmatrix}0\\0\\1\end{pmatrix} = \alpha_0 \psi_0 + \alpha_+ \psi_+ + \alpha_- \psi_-$

$\langle \psi_0 | \begin{smallmatrix}0\\0\\1\end{smallmatrix} \rangle = 0$ $\quad \langle \psi_+ | \begin{smallmatrix}0\\0\\1\end{smallmatrix} \rangle = +\frac{1}{\sqrt{2}}$ $\quad \langle \psi_- | \begin{smallmatrix}0\\0\\1\end{smallmatrix} \rangle = -\frac{1}{\sqrt{2}}$

future: $\frac{1}{\sqrt{2}} \psi_+ e^{-i(a+b)t/\hbar} + \left(\frac{-1}{\sqrt{2}}\right) \psi_- e^{-i(a-b)t/\hbar}$

3-38

$$\begin{array}{lccc} H: \text{ eigen values} & \hbar\omega & 2\hbar\omega & 2\hbar\omega \\ \text{vectors} & \begin{pmatrix}1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\end{pmatrix} \end{array}$$

$$\begin{array}{lccc} A: \text{ eigen values} & \lambda & -\lambda & 2\lambda \\ \text{vectors} & \begin{pmatrix}1\\1\\0\end{pmatrix} & \begin{pmatrix}1\\-1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\end{pmatrix} \\ & \hookrightarrow \frac{1}{\sqrt{2}} & \hookrightarrow \frac{1}{\sqrt{2}} & \end{array}$$

$$\begin{array}{lccc} B \text{ eigen values} & 2\mu & \mu & -\mu \\ \text{vectors} & \begin{pmatrix}1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\1\end{pmatrix} & \begin{pmatrix}0\\1\\-1\end{pmatrix} \\ & & \hookrightarrow \frac{1}{\sqrt{2}} & \hookrightarrow \frac{1}{\sqrt{2}} \end{array}$$

$\langle H \rangle = (c_1^* \; c_2^* \; c_3^*) \begin{pmatrix} 1 & & \\ & 2 & \\ & & 2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \hbar\omega = \hbar\omega \left[1|c_1|^2 + 2|c_2|^2 + 2|c_2|^2 \right]$

$\langle A \rangle \qquad \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \lambda = \lambda \left[c_1^* c_2 + c_2^* c_1 + 2|c_3|^2 \right]$

$\langle B \rangle \qquad \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \mu = \mu \left[2|c_1|^2 + c_2^* c_3 + c_3^* c_2 \right]$

$|c_i|^2$ is prob of measuring E_i $\qquad |S\rangle = c_1 \begin{pmatrix}1\\0\\0\end{pmatrix} e^{-i\omega t} + c_2 \begin{pmatrix}0\\1\\0\end{pmatrix} e^{-2i\omega t} + c_3 \begin{pmatrix}0\\0\\1\end{pmatrix} e^{-2\omega}$

To find, say the prob a measurement of B yields $-\mu$, dot $|S\rangle$ with

$$\frac{1}{\sqrt{2}} \begin{pmatrix}0\\1\\-1\end{pmatrix} \cdot \left\{ c_1 \begin{pmatrix}1\\0\\0\end{pmatrix} e^{-i\omega t} + c_2 \begin{pmatrix}0\\1\\0\end{pmatrix} e^{-2i\omega t} + c_3 \begin{pmatrix}0\\0\\1\end{pmatrix} e^{-2i\omega t} \right\}$$

$$= \frac{1}{\sqrt{2}} (c_2 - c_3) e^{-2i\omega t}$$

$$\text{Prob} = |\quad|^2 = \frac{1}{2} (c_2 - c_3)^2$$

3-39 $$e^{i x_0 \frac{p}{\hbar}} f = e^{\overbrace{x_0 \partial_x}^{1 + x_0\partial_x + \frac{1}{2!}x_0^2\partial_x^2 + \frac{1}{3!}x_0^3\partial_x^2}} f(x) = f(x) + x_0 f'(x) + \frac{1}{2}x_0^2 f''(x) +$$

$$= f(x+x_0)$$ by Taylor Expansion

$$e^{-i t_0 H/\hbar}\psi = e^{+t_0\partial_t}\psi \quad \checkmark$$

online Commutators

$$[p, x^2]f = p x^2 f - x^2 p f = \frac{\hbar}{i}2x f + x^2 p f - x^2 p f = \frac{\hbar}{i}2x$$

$$[p, V]f = p V f - V p f = \frac{\hbar}{i}V' f + V p f - V p f = \frac{\hbar}{i}V' f$$

$$\left.\begin{aligned}[p^2, x] &= p[p,x] + [p,x]p = 2\frac{\hbar}{i}p \\ [p,x] &= \frac{\hbar}{i}p^0\end{aligned}\right\} \text{cases } n = 1 \,\&\, 2$$

To show case $n \Rightarrow$ case $n+1$

$$[p^n, x] = p[p^{n-1}, x] + [p,x]p^{n-1} = p\,(n-1)\frac{\hbar}{i}p^{n-2} + \frac{\hbar}{i}p^{n-1}$$

$$= n\frac{\hbar}{i}p^{n-1} \quad \checkmark$$

3-31 $$[\underbrace{\tfrac{p^2}{2m} + V}_{H}, xp] = \frac{1}{2m}\{[p^2, xp]\} + [V, xp]$$

$$[p^2, xp] \downarrow \; \underbrace{[p^2, x]}_{2\frac{\hbar}{i}p}p + x\underbrace{[p^2, p]}_{\hookrightarrow 0}$$

$$[V, xp] \hookrightarrow \underbrace{[V,x]}_{\downarrow 0}p + x\underbrace{[V,p]}_{\hookrightarrow -\frac{\hbar}{i}V'}$$

$$= \frac{\hbar}{mi}xp^2 - \frac{\hbar}{i}xV'$$

$$\frac{d}{dt}\langle xp\rangle = \langle \overset{0}{\frac{\partial xp}{\partial t}}\rangle + \langle \tfrac{1}{m}p^2 - xV'\rangle = \langle 2T\rangle - \langle xV'\rangle$$

$$\frac{p^2}{m} \downarrow 2T$$

stationary state $\Rightarrow \psi = e^{-iEt/\hbar}$ so $\psi^*\psi \sim e^0$ so $\frac{d}{dt}\langle\,\rangle = 0$

For SHO: $V = \frac{1}{2}kx^2 \quad xV' = kx^2 = 2V$