$$\int_{-\infty}^{+\infty} e^{-\alpha x^2-\beta x}\,dx = \sqrt{\frac{\pi}{\alpha}}\,e^{\beta^2/4\alpha} \qquad \int_0^{+\infty} x\,e^{-\alpha x^2}\,dx = \frac{1}{2\alpha} \qquad \int_0^{\infty} x^n\,e^{-\alpha x}\,dx = n!/\alpha^{n+1} \qquad \int f(x)\delta(x-a)\,dx = f(a)$$

$$H\psi = i\hbar\partial_t\psi \qquad H\psi = E\psi \qquad H = \frac{p^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\,\partial_x^2 + V(x) \qquad p = -i\hbar\partial_x \qquad [p,x] = -i\hbar$$

$\partial_t\;\psi^*(x,t)\psi(x,t) = -\partial_x J$ where current $J = \frac{\hbar}{2im}\left(\psi^*\,\partial_x\psi - \psi\,\partial_x\psi^*\right) = \frac{\hbar}{2im}\;\psi^*\;\overset{\leftrightarrow}{\partial_x}\;\psi$ probability density= $\psi^*\psi$

Particle-in-a-box with $V(x) = 0$ for $0 < x < L$, but $V(x) = \infty$ elsewhere

$$E_n = \frac{(\hbar k)^2}{2m} \qquad u_n(x) = \sqrt{\frac{2}{L}}\,\sin(kx) \quad \text{where} \quad k = \frac{n\pi}{L} \qquad n = 1, 2, 3\ldots$$

3-d: $|n_x n_y n_z\rangle = u_{n_x}(x)u_{n_y}(y)u_{n_z}(z)$ $\quad E = \frac{(\hbar k)^2}{2m}$ where $\vec{\mathbf{k}} = (n_x\pi/L_x, n_y\pi/L_y, n_z\pi/L_z)$

spherical: $R(r) = j_\ell(kr)$ $\qquad E = \frac{(\hbar k)^2}{2m}$ $\qquad$ in box: $kR =$ zero of j_ℓ

Harmonic Oscillator with $V(x) = \frac{1}{2}m\omega^2x^2$ $\qquad E_n = \hbar\omega\,(n + \tfrac{1}{2})$ $\qquad n = 0, 1, 2\ldots$

$|n\rangle = u_n(x) = N_n H_n(\xi)\,e^{-\frac{1}{2}\xi^2}$ $\quad$ where $\quad \xi = \sqrt{\frac{m\omega}{\hbar}}\,x$ $\quad$ and H_n is an n^{th} degree polynomial

$$a_- = \sqrt{\frac{m\omega}{2\hbar}}\left(x + i\frac{p}{m\omega}\right) = \frac{1}{\sqrt{2}}\,(\xi + \partial_\xi) \qquad a_+ = a_-^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(x - i\frac{p}{m\omega}\right) = \frac{1}{\sqrt{2}}\,(\xi - \partial_\xi) \qquad x = \sqrt{\frac{\hbar}{2m\omega}}\,(a_+ + a_-)$$

$$[a_-, a_+] = 1 \qquad [H, a_\pm] = \pm\hbar\omega a_\pm \qquad H = \hbar\omega\left(\tfrac{1}{2} + a_+a_-\right) \qquad a_-\,|n\rangle = \sqrt{n}\,|n-1\rangle \qquad a_+\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

2-d: $(n_x + n_y + 1) = (2n_r + |m| + 1)$ $\qquad$ 3-d: $\left(n_x + n_y + n_z + \tfrac{3}{2}\right) = \left(2n_r + \ell + \tfrac{3}{2}\right)$

Angular Momentum: $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$ $\qquad [L_i, V_j] = i\hbar\epsilon_{ijk}V_k$ where vector $\vec{\mathbf{V}} = \vec{\mathbf{r}}, \vec{\mathbf{p}}, \vec{\mathbf{L}}$ $\qquad |\ell m\rangle = Y_{\ell m}(\theta,\phi)$ $\qquad -\ell \le m \le +\ell$

$$\vec{\mathbf{L}}^2\,|\ell m\rangle = \ell(\ell+1)\,\hbar^2\,|\ell m\rangle \qquad L_z\,|\ell m\rangle = m\hbar\,|\ell m\rangle \qquad L_\pm\,|\ell m\rangle = \sqrt{\ell(\ell+1) - m(m\pm 1)}\;\hbar\,|\ell\, m\pm 1\rangle$$

$$L_\pm = L_x \pm iL_y \qquad [L_+, L_-] = 2\hbar L_z \qquad [L_z, L_\pm] = \pm\hbar L_\pm \qquad [\vec{\mathbf{L}}^2, L_\pm] = 0 \qquad [L_i, \vec{\mathbf{V}}\cdot\vec{\mathbf{W}}] = 0$$

Spin $\frac{1}{2}$: $\quad \vec{\mathbf{S}} = \frac{\hbar}{2}\,\vec{\sigma}$ $\qquad |\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \chi_+ = \uparrow = \begin{pmatrix}1\\0\end{pmatrix}$ $\qquad |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \chi_- = \downarrow = \begin{pmatrix}0\\1\end{pmatrix}$

Clebsch-Gordan: $|jm\rangle = \sum C(jm;\ell m_\ell, sm_s)\,|\ell m_\ell\rangle\,|sm_s\rangle$ $\qquad$ know how to use table!

Radial Equation: $\psi(r,\theta,\phi) = Y_{\ell m}(\theta,\phi)R(r)$ $\qquad R(r) = \frac{u(r)}{r}$

$$\left[\frac{-\hbar^2}{2m}\left(\partial_r^2 + \frac{2}{r}\partial_r\right) + \frac{\hbar^2\ell(\ell+1)}{2mr^2} + V(r)\right]R = E\,R \qquad \left[\frac{-\hbar^2}{2m}\partial_r^2 + \frac{\hbar^2\ell(\ell+1)}{2mr^2} + V(r)\right]u = E\,u$$

H atom: $\quad H = \frac{p^2}{2m} - \frac{Ze^2}{4\pi\epsilon_0 r}$ $\qquad E_n = -\frac{1}{2}\,mc^2\,\frac{(Z\alpha)^2}{n^2} = -\frac{1}{2}\,\frac{Z^2e^2}{4\pi\epsilon_0 a_0 n^2} \approx -13.6\text{ eV}\;\frac{Z^2}{n^2}$ $\qquad n = 1, 2, 3, \ldots$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2} \approx .53\text{ Å} \qquad \alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137} \qquad n = n_r + \ell + 1\;\therefore\;0 \le \ell \le n-1 \qquad \rho = \sqrt{\frac{8m|E|}{\hbar^2}}\,r = \frac{2Zr}{na_0}$$

$|n\ell m\rangle = R_{n\ell}(\rho)\,Y_{\ell m}(\theta,\phi)$ $\quad$ where $\quad R_{n\ell} = N_{n\ell}\,\rho^\ell\,L_{n_r}^{2\ell+1}(\rho)\,e^{-\frac{1}{2}\rho}$ $\qquad N_{n\ell} = \frac{2}{n^2}\sqrt{\frac{(n-\ell-1)!}{(n+\ell)!}}$

Spectroscopic Notation: orbital: s,p,d,f,g $\quad$ term: ${}^{2S+1}L_J$ $\quad$ atomic: 1s,2s,2p,3s,3p,4s,3d $\quad$ nuclear: 1s,1p,1d,2s,1f,2p,1g

2-particle CM Coordinates:

$$\begin{array}{lll} \vec{\mathbf{R}} = \frac{m_1}{M}\,\vec{\mathbf{r}}_1 + \frac{m_2}{M}\,\vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_1 = \vec{\mathbf{R}} + \frac{m_2}{M}\,\vec{\mathbf{r}} & M = m_1 + m_2 \\ \vec{\mathbf{r}} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2 & \vec{\mathbf{r}}_2 = \vec{\mathbf{R}} - \frac{m_1}{M}\,\vec{\mathbf{r}} & \mu = \frac{m_1m_2}{M} \end{array} \qquad \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{P^2}{2M} + \frac{p^2}{2\mu}$$

Magnetic: $\vec{\mathbf{p}} \to \vec{\mathbf{p}} - q\vec{\mathbf{A}}$ $\qquad$ where q is charge, e.g., for electron: $q = -e$ $\qquad \vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$ $\qquad$ e.g., uniform $\vec{\mathbf{B}}$ from $\vec{\mathbf{A}} = \frac{1}{2}\,\vec{\mathbf{B}} \times \vec{\mathbf{r}}$

Spin-statistics: $\qquad$ fermion: $s = \frac{1}{2}, \frac{3}{2}, \ldots$ $\qquad$ boson: $s = 0, 1, 2, \ldots$

$$u(x_i) = (N!)^{-1/2}\begin{vmatrix} f(x_1) & f(x_2) & f(x_3) & \cdots \\ g(x_1) & g(x_2) & g(x_3) & \cdots \\ h(x_1) & h(x_2) & h(x_3) & \cdots \\ \vdots & \vdots & \vdots & \end{vmatrix}$$