```
|lm>=|m>
|1/2 1/2>=|u>; |1/2 -1/2>=|d>

product wavefunctions:
|lm> |1/2 1/2> = |m>|u>= |m u>
|lm+1> |1/2 -1/2> = |m+1>|d>= |m+1 d>

L+ |lm> = Sqrt[l(l+1)-m(m+1)] |lm+1>
L- |lm> = Sqrt[l(l+1)-m(m-1)] |lm-1>
S+ |u>=0; S+ |d>=|u>; S- |u>=|d>; S- |d>=0;

L+ S- |m u> = Sqrt[l(l+1)-m(m+1)] |m+1 d>
L+ S- |m+1 d> = 0

L- S+ |m u> = 0
L- S+ |m+1 d> = Sqrt[l(l+1)-(m+1)m] |m u>

Lz Sz |m u> = m/2 |m u>
Lz Sz |m+1 d> = -(m+1)/2 |m u>

(L^2+S^2) X = (l(l+1)+3/4) X

J2={{(l(l+1)+3/4)+m, Sqrt[l(l+1)-(m+1)m] },
{Sqrt[l(l+1)-m(m+1)], (l(l+1)+3/4)-(m+1)}}

Eigensystem[J2]
                    2                2
           -1 + 4 l   3 + 8 l + 4 l
Out[13]= {{---------, --------------},
               4            4

                     2        2                     2        2
         Sqrt[l + l  - m - m ]        Sqrt[l + l  - m - m ]
>    {{-(---------------------), 1}, {---------------------, 1}}}
               1 + l + m                      l - m

Look carefully at signs: v1 has mixed signs; v2 is both postive.

values=Factor[First[%]]
          (-1 + 2 l) (1 + 2 l)  (1 + 2 l) (3 + 2 l)
Out[14]= {--------------------, -------------------}
                   4                     4

these are j(j+1) for j=l-1/2 amd j=l+1/2

vectors= {Normalize[First[Last[%%]]],Normalize[Last[Last[%%]]]}
v2=Simplify[vectors^2,Assumptions->{l>m,m>0}]

           l - m   1 + l + m    1 + l + m   l - m
Out[5]= {{-------, ---------}, {---------, -------}}
         1 + 2 l   1 + 2 l      1 + 2 l   1 + 2 l

%5 /. {l->3/2,m->-3/2}
          3  1    1  3
Out[6]= {{-, -}, {-, -}}
          4  4    4  4

%5 /. {l->2,m->1}
          1  4    4  1
Out[7]= {{-, -}, {-, -}}
          5  5    5  5
```