

```
|lm>=|m>
|1 1>=|u>; |1 0>=|s>; |1 -1>=|d>
```

product wavefunctions:

```
|lm-1> |1 1> = |m-1>|u>= |m-1 u>
|lm> |1 0 > = |m>|s>= |m s>
|lm+1> |1 -1> = |m+1>|d>= |m+1 d>
```

```
L+ |lm> = Sqrt[1(1+1)-m(m+1)] |lm+1>
L- |lm> = Sqrt[1(1+1)-m(m-1)] |lm-1>
S+ |u>=0; S+ |s>=Sqrt[2] |u>; S+ |d>=Sqrt[2] |u>;
S- |u>=Sqrt[2] |s>; S- |s>=Sqrt[2] |d>; S- |d>=0
```

```
L+ S- |m-1 u> = Sqrt[1(1+1)-m(m-1)] Sqrt[2] |m s>
L+ S- |m s> = Sqrt[1(1+1)-m(m+1)] Sqrt[2] |m+1 d>
L+ S- |m+1 d> = 0
```

```
L- S+ |m-1 u> = 0
L- S+ |m s> = Sqrt[1(1+1)-m(m-1)] Sqrt[2] |m-1 u>
L- S+ |m+1 d> = Sqrt[1(1+1)-m(m+1)] Sqrt[2] |m s>
```

```
Lz Sz |m-1 u> = (m-1) |m-1 u>
Lz Sz |m s> = 0
Lz Sz |m+1 d> = -(m+1) |m+1 d>
```

```
(L^2+S^2) X = (1(1+1)+2) X
```

```
J2={{(1(1+1)+2)+2(m-1),Sqrt[1(1+1)-m(m-1)] Sqrt[2],0},
{Sqrt[1(1+1)-m(m-1)] Sqrt[2],(1(1+1)+2),Sqrt[1(1+1)-m(m+1)] Sqrt[2]},
{0, Sqrt[1(1+1)-m(m+1)] Sqrt[2],(1(1+1)+2)-2(m+1)}}
```

Eigensystem[J2]

```
Out[18]= {{{(-1 + 1) 1, 1 (1 + 1), 2 + 3 1 + 1},
           {
             Sqrt[1 + 1 - m - m] Sqrt[1 + 1 + m - m]
           } / {
             (1 + m) (1 + 1 + m)
           },
           {
             Sqrt[2] Sqrt[1 + 1 - m - m]
           } / {
             1 + 1 + m
           }, 1},
           {
             Sqrt[1 + 1 + m - m] Sqrt[2] m Sqrt[1 + 1 - m - m]
           } / {
             Sqrt[1 + 1 - m - m] -1 - 1 + m + m
           }, 1},
           {
             Sqrt[1 + 1 - m - m] Sqrt[1 + 1 + m - m]
           } / {
             1 + 1 - m - 2 1 m + m
           },
           {
             Sqrt[2] (-2 - 2 1 + 2 m) Sqrt[1 (1 + 1) - m (1 + m)]
           } / {
             2 1 + 2 1 - 2 m - 4 1 m + 2 m
           }, 1}}}
```

Look carefully at signs: v1=(+,-,+), v2=(-,+,+), v3=(+,+,+) has mixed signs; v2 is both positive.

```
values=Factor[First[%]]
```

```
Out[19]= {(-1 + 1) 1, 1 (1 + 1), (1 + 1) (2 + 1)}
```

```
vectors={Normalize[First[Last[%]]],Normalize[Last[%]][[2]],Normalize[Last[Last[%]]]}
v2=Factor[Simplify[vectors^2,Assumptions->{l>m+1,m>0}]}
```

```
Out[33]= {{{
             (1 - m) (1 + 1 - m) (1 - m) (1 + m) (1 + m) (1 + 1 + m)
           } / {
             2 1 (1 + 2 1) 1 (1 + 2 1) 2 1 (1 + 2 1)
           },
           {
             (1 + 1 - m) (1 + m) m (1 - m) (1 + 1 + m)
           } / {
             2 1 (1 + 1) 1 (1 + 1) 2 1 (1 + 1)
           },
           {
             (1 + m) (1 + 1 + m) (1 + 1 - m) (1 + 1 + m) (1 - m) (1 + 1 - m)
           } / {
             2 (1 + 1) (1 + 2 1) (1 + 1) (1 + 2 1) 2 (1 + 1) (1 + 2 1)
           }}}
```

```
% /. {1->2,m->1}
```

```
Out[34]= {{{-, -, -}, {-, -, -}, {-, -, -}}
```