```
|lm>=|m>
|1/2 1/2>=|u>; |1/2 -1/2>=|d>

product wavefunctions:
|lm> |1/2 1/2> = |m>|u>= |m u>
|lm+1> |1/2 -1/2> = |m+1>|d>= |m+1 d>

L+ |lm> = Sqrt[l(l+1)-m(m+1)] |lm+1>
L- |lm> = Sqrt[l(l+1)-m(m-1)] |lm-1>
S+ |u>=0; S+ |d>=|u>; S- |u>=|d>; S- |d>=0;

L+ S- |m u> = Sqrt[l(l+1)-m(m+1)] |m+1 d>
L+ S- |m+1 d> = 0

L- S+ |m u> = 0
L- S+ |m+1 d> = Sqrt[l(l+1)-(m+1)m] |m u>

Lz Sz |m u> = m/2 |m u>
Lz Sz |m+1 d> = -(m+1)/2 |m u>

(L^2+S^2) X = (l(l+1)+3/4) X

J2={{(l(l+1)+3/4)+m, Sqrt[l(l+1)-(m+1)m] },
{Sqrt[l(l+1)-m(m+1)], (l(l+1)+3/4)-(m+1)}}

Eigensystem[J2]
                    2                2
           -1 + 4 l   3 + 8 l + 4 l
Out[13]= {{---------, --------------},
               4            4

                     2        2                     2        2
         Sqrt[l + l  - m - m ]        Sqrt[l + l  - m - m ]
>    {{-(---------------------), 1}, {---------------------, 1}}}
               1 + l + m                      l - m

Look carefully at signs: v1 has mixed signs; v2 is both postive.

values=Factor[First[%]]
          (-1 + 2 l) (1 + 2 l)  (1 + 2 l) (3 + 2 l)
Out[14]= {--------------------, -------------------}
                   4                     4

these are j(j+1) for j=l-1/2 amd j=l+1/2

vectors= {Normalize[First[Last[%%]]],Normalize[Last[Last[%%]]]}
v2=Simplify[vectors^2,Assumptions->{l>m,m>0}]

           l - m   1 + l + m    1 + l + m   l - m
Out[5]= {{-------, ---------}, {---------, -------}}
         1 + 2 l   1 + 2 l      1 + 2 l   1 + 2 l

%5 /. {l->3/2,m->-3/2}
          3  1    1  3
Out[6]= {{-, -}, {-, -}}
          4  4    4  4

%5 /. {l->2,m->1}
          1  4    4  1
Out[7]= {{-, -}, {-, -}}
          5  5    5  5
```

```
|lm>=|m>
|1 1>=|u>; |1 0>=|s>; |1 -1>=|d>

product wavefunctions:
|lm-1> |1  1> = |m-1>|u>= |m-1 u>
|lm> |1  0 >  = |m>|s>= |m s>
|lm+1> |1 -1> = |m+1>|d>= |m+1 d>

L+ |lm> = Sqrt[l(l+1)-m(m+1)] |lm+1>
L- |lm> = Sqrt[l(l+1)-m(m-1)] |lm-1>
S+ |u>=0; S+ |s>=Sqrt[2] |u>; S+ |d>=Sqrt[2] |u>;
S- |u>=Sqrt[2] |s>; S- |s>=Sqrt[2] |d>; S- |d>=0

L+ S- |m-1 u> = Sqrt[l(l+1)-m(m-1)] Sqrt[2] |m s>
L+ S- |m s> = Sqrt[l(l+1)-m(m+1)] Sqrt[2] |m+1 d>
L+ S- |m+1 d> = 0

L- S+ |m-1 u> = 0
L- S+ |m s> = Sqrt[l(l+1)-m(m-1)] Sqrt[2] |m-1 u>
L- S+ |m+1 d> = Sqrt[l(l+1)-m(m+1)] Sqrt[2] |m s>

Lz Sz |m-1 u> = (m-1) |m-1 u>
Lz Sz |m s> = 0
Lz Sz |m+1 d> = -(m+1) |m+1 d>

(L^2+S^2) X = (l(l+1)+2) X

J2={{(l(l+1)+2)+2(m-1),Sqrt[l(l+1)-m(m-1)] Sqrt[2],0},
{Sqrt[l(l+1)-m(m-1)] Sqrt[2],(l(l+1)+2),Sqrt[l(l+1)-m(m+1)] Sqrt[2]},
{0, Sqrt[l(l+1)-m(m+1)] Sqrt[2],(l(l+1)+2)-2(m+1)}}

Eigensystem[J2]
                                          2
Out[18]= {{(-1 + l) l, l (1 + l), 2 + 3 l + l },

                 2        2            2        2
       Sqrt[l + l  - m - m ] Sqrt[l + l  + m - m ]
>    {{-----------------------------------------,
                  (l + m) (1 + l + m)

                         2        2
         Sqrt[2] Sqrt[l + l  - m - m ]
>      -(----------------------------), 1},
                   1 + l + m

                   2        2                          2        2
         Sqrt[l + l  + m - m ]     Sqrt[2] m Sqrt[l + l  - m - m ]
>     {-(---------------------), -(------------------------------), 1},
                   2        2                 2        2
         Sqrt[l + l  - m - m ]            -l - l  + m + m

                 2        2            2        2
       Sqrt[l + l  - m - m ] Sqrt[l + l  + m - m ]
>     {-----------------------------------------,
                    2               2
                 l + l  - m - 2 l m + m

         Sqrt[2] (-2 - 2 l + 2 m) Sqrt[l (1 + l) - m (1 + m)]
>      -(-------------------------------------------------), 1}}}
                          2                     2
                   2 l + 2 l  - 2 m - 4 l m + 2 m

Look carefully at signs: v1=(+,-,+), v2=(-,+,+), v3=(+,+,+) has mixed signs; v2 is both postive.

values=Factor[First[%]]
Out[19]= {(-1 + l) l, l (1 + l), (1 + l) (2 + l)}

vectors= {Normalize[First[Last[%%]]],Normalize[Last[%%][[2]]],Normalize[Last[Last[%%]]]}
v2=Factor[Simplify[vectors^2,Assumptions->{l>m+1,m>0}]]

            (l - m) (1 + l - m)  (l - m) (l + m)  (l + m) (1 + l + m)
Out[33]= {{-------------------, ---------------, -------------------},
                2 l (1 + 2 l)       l (1 + 2 l)       2 l (1 + 2 l)

                              2
       (1 + l - m) (l + m)    m       (l - m) (1 + l + m)
>    {-------------------, ---------, -------------------},
            2 l (1 + l)      l (1 + l)      2 l (1 + l)

       (l + m) (1 + l + m)  (1 + l - m) (1 + l + m)  (l - m) (1 + l - m)
>    {-------------------, ----------------------, -------------------}}
        2 (1 + l) (1 + 2 l)     (1 + l) (1 + 2 l)     2 (1 + l) (1 + 2 l)

% /. {l->2,m->1}
            1   3   3    1  1  1    2  8   1
Out[34]= {{--, --, -}, {-, -, -}, {-, --, --}}
            10  10  5    2  6  3    5  15  15
```