A solvable (but unrealistic) special case

$$V_{ab} = \tilde{V}_{ab}\tfrac{1}{2}e^{i\omega t} \qquad \text{Note } \tilde{V}_{ba} = \tilde{V}_{ab}^{*}$$

$$V_{ba} = \tilde{V}_{ba}\tfrac{1}{2}e^{-i\omega t}$$

exact eqns:

$$\dot{c}_a = \frac{1}{i\hbar}\frac{\tilde{V}_{ab}}{2} e^{i\omega t} e^{-i\omega_0 t} c_b = \frac{\tilde{V}_{ab}}{2i\hbar} e^{-i\overbrace{(\omega_0-\omega)}^{\alpha \text{ think small}}t} c_b$$

$$\dot{c}_b = \frac{1}{i\hbar}\frac{\tilde{V}_{ba}}{2} e^{-i\omega t} e^{+i\omega_0 t} c_a = \frac{\tilde{V}_{ba}}{2i\hbar} e^{+i\alpha t} c_a$$

$\longleftarrow$

$$\ddot{c}_a = -i\alpha\dot{c}_a + \frac{|\tilde{V}|^2}{-4\hbar^2} c_a$$

$$\ddot{c}_a + i\alpha\dot{c}_a + \frac{\beta^2}{4}c_a = 0 \qquad c_a = e^{i\gamma t}$$

$$-\gamma^2 - \alpha\gamma + \frac{\beta^2}{4} = 0 \quad ; \quad \gamma^2 + \alpha\gamma - \frac{\beta^2}{4} = 0$$

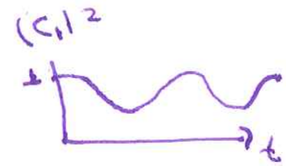
$$\gamma = \frac{-\alpha \pm \sqrt{\alpha^2 + \beta^2}}{2}$$

$$c_a = A e^{i\gamma_+ t} + B e^{i\gamma_- t}$$

$$\dot{c}_a = i\gamma_+ A e^{i\gamma_+ t} + i\gamma_- B e^{i\gamma_- t} = 0 \text{ @ } t = 0$$

$$A + B = 1$$

$$A\gamma_+ + B\gamma_- = 0 = -\frac{\alpha}{2}(A+B) + \frac{\Gamma}{2}(A-B)$$



$$A - B = \frac{\alpha}{\Gamma}$$

$$A = \tfrac{1}{2}\left(1 + \frac{\alpha}{\Gamma}\right)$$

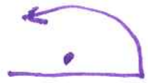
$$B = \tfrac{1}{2}\left(1 - \frac{\alpha}{\Gamma}\right)$$

$$|c_a|^2 = \cos^2 + \left(\frac{\alpha}{\Gamma}\right)^2 \sin^2$$
$$= 1 - \left(1 - \left(\frac{\alpha}{\Gamma}\right)^2\right)\sin^2$$

$$c_a = e^{-\frac{i\alpha}{2}t}\left\{\cos\left(\frac{\Gamma}{2}t\right) + i\frac{\alpha}{\Gamma}\sin\left(\frac{\Gamma}{2}t\right)\right\}$$

Note: $\frac{d}{dt}(c_a c_a^* + c_b c_b^*) = \underset{\downarrow\, c_b}{\dot{c}_a} c_a^* + c_a \underset{\downarrow\, -c_b^*}{\dot{c}_a^*} + \underset{\downarrow\, c_a}{\dot{c}_b} c_b + c_b \underset{\downarrow\, -c_a^*}{\dot{c}_b^*} = 0$

$$\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2}\,dx = \frac{-1}{4}\int_{-\infty}^{\infty} \frac{(e^{ix}-e^{-ix})^2}{x^2+\epsilon^2} = -\frac{1}{4}\int_{-\infty}^{\infty} \underbrace{\frac{(e^{2ix}-1)}{x^2+\epsilon^2}}_{A} + \underbrace{\frac{(e^{-2ix}-1)}{x^2+\epsilon^2}}_{B}$$

(A)  $2\pi i\left[\frac{2i(i\epsilon)}{2i\epsilon}\right] = -2\pi$

(B)  $-2\pi i\left[\frac{-2i(-i\epsilon)}{-2i\epsilon}\right] = -2\pi$

$$\left.\begin{matrix} -2\pi \\ -2\pi \end{matrix}\right\} -\frac{1}{4}(-4\pi) = \pi$$

Area under $\int \frac{\sin^2[\Xi t]}{\Xi^2 t^2}\, d\Xi\, t^2 = \pi t$

$$|c_b|^2 = \frac{|V_{ba}|^2}{4\hbar^2}\frac{\sin^2 \Xi t}{\Xi^2} \quad \text{where } \Xi = \frac{\omega_0-\omega}{2}$$

electric dipole p

For electric field → $V = q(-E_0 z) = -E_0\,(qz)$ ← $q=-e$

energy density = $\frac{\text{Energy}}{\text{Volume}} = u = \frac{\epsilon_0}{2}E_0^2$ ← includes B_0^2; time average

u ← $\rho(\omega)\,d\omega$

Assume lots of different freq/polarizations simultaneously present & use incoherent sum

Note: $d\omega = 2\,d\Xi$

$$P_{a\to b} = \frac{E_0^2|\vec p|^2}{4\hbar^2}\frac{\sin^2\Xi t}{\Xi^2} \to \frac{|\vec p|^2}{\hbar^2\epsilon_0}\overbrace{\int \rho\, \frac{\sin^2 \Xi t}{\Xi^2}\,d\Xi}^{\rho(\omega_0)\pi t}$$

Now $P_{a\to b}$ = $P_{b\to a}$

E_0^2 ← $\rho\, d\omega\, \frac{2}{\epsilon_0}$; $d\omega$ ← $2\,d\Xi$

seek avg value for all possible polarizations

put $\vec p$ on z axis: $\vec p\cdot\hat n = \cos\theta$

$$\frac{\iint\cos^2\theta\sin\theta\, d\theta\, d\phi}{\iint \sin\theta\, d\theta\, d\phi} = \frac{1}{3}$$

$$\text{Rate} = \frac{\pi|\vec p|^2}{3\epsilon_0\hbar^2}\rho(\omega_0)$$