Time dependent perturbation theory $\mathbb{H} = H + V$ ← depends on t

$H\psi_i = E_i \psi_i$ ← complete ; $\omega_i \equiv E_i/\hbar$

Assume 2 levels : a & b $\Psi = c_a \psi_a e^{-i\omega_a t} + c_b \psi_b e^{-i\omega_b t}$

"transitions"

Note: since Ch2 we've had similar expressions but then cs were constant now c depends on time → transitions

$$\mathbb{H}\Psi = c_a E_a \psi_a e^{-i\omega_a t} + c_a V \psi_a e^{-i\omega_a t} + c_b E_b \psi_b e^{-i\omega_b t} + c_b V \psi_b e^{-i\omega_b t}$$

$$i\hbar \partial_t \Psi = i\hbar \dot{c}_a \psi_a e^{-i\omega_a t} + c_a E_a \psi_a e^{-i\omega_a t} + i\hbar \dot{c}_b \psi_b e^{-i\omega_b t} + c_b E_b \psi_b e^{-i\omega_b t}$$

$$c_a V \psi_a e^{-i\omega_a t} + c_b V \psi_b e^{-i\omega_b t} = i\hbar \left\{ \dot{c}_a \psi_a e^{-i\omega_a t} + \dot{c}_b \psi_b e^{-i\omega_b t} \right\}$$

$\langle \psi_a | \Rightarrow$ $c_a V_{aa} e^{-i\omega_a t} + c_b V_{ab} e^{-i\omega_b t} = i\hbar \dot{c}_a e^{-i\omega_a t}$

$\boxed{\omega_0 \equiv \omega_b - \omega_a}$ $c_a V_{aa} + c_b V_{ab} e^{-i\omega_0 t} = i\hbar \dot{c}_a$

$\langle \psi_b | \Rightarrow$ $c_a V_{ba} e^{-i\omega_a t} + c_b V_{bb} e^{-i\omega_b t} = i\hbar \dot{c}_b e^{-i\omega_b t}$

$$c_a V_{ba} e^{+i\omega_0 t} + c_b V_{bb} = i\hbar \dot{c}_b$$

assume $V_{aa} = V_{bb} = 0$

this strong assumption is most often the actual case

$$\dot{c}_a = \frac{1}{i\hbar} c_b V_{ab} e^{-i\omega_0 t}$$

$$\dot{c}_b = \frac{1}{i\hbar} c_a V_{ba} e^{+i\omega_0 t}$$

Assume $c_a = 1$ $c_b = 0$

$$\boxed{c_b = \frac{1}{i\hbar} \int^t V_{ba} e^{+i\omega_0 t} dt}$$

iterate.

$$\dot{c}_a = \frac{-1}{\hbar^2} \int^t V_{ba} e^{+i\omega_0 t'} dt' \, V_{ab} e^{-i\omega_0 t}$$

$$c_a = 1 - \frac{1}{\hbar^2} \int^t \int^t V_{ba} e^{+i\omega_0 t'} dt' \, V_{ab} e^{-i\omega_0 t} dt$$

Note: if had multiple states result for "b" stands result for "a" would involve sum.

Sinusoidal perturbation (eq Light) $V_{ab} = \tilde{V}_{ab}\cos(\omega t)$

note: $\omega \sim 10^{15}\,Hz$

does not depend on time

$$c_b = \frac{1}{i\hbar}\int^t \tilde{V}_{ba}\underbrace{\left(\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right)}_{\cos(\omega t)} e^{+i\omega_0 t}\,dt$$

$$= \frac{\tilde{V}_{ba}}{2i\hbar}\left\{\int_0^t e^{i(\omega+\omega_0)t} + e^{i(\omega_0-\omega)t}\right\}dt$$

$$\frac{e^{i(\omega+\omega_0)t}\Big|_0^t}{i(\omega+\omega_0)} \qquad \frac{e^{i(\omega_0-\omega)t}\Big|_0^t}{i(\omega_0-\omega)}$$

small neglect

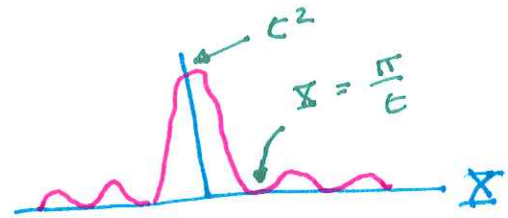
perhaps big if $\omega_0 \approx \omega$

$$= \frac{\tilde{V}_{ba}}{i\hbar}\left\{\frac{e^{i(\omega_0-\omega)t}-1}{2i(\omega_0-\omega)}\right\} \quad \left[e^{i\left(\frac{\omega_0-\omega}{2}\right)t} - e^{-i\left(\frac{\omega_0-\omega}{2}\right)t}\right]e^{i\left(\frac{\omega_0-\omega}{2}\right)t}$$

$$= \frac{\tilde{V}_{ba}}{2i\hbar}\,\frac{\sin\left[\frac{\omega_0-\omega}{2}t\right]}{\frac{(\omega_0-\omega)}{2}}\,e^{i\left(\frac{\omega_0-\omega}{2}t\right)}$$

$$X = \frac{\omega_0-\omega}{2}$$

$$|c_b|^2 = \frac{\tilde{V}_{ba}}{4\hbar^2}\,\frac{\sin^2[Xt]}{X^2}$$



note $X = 0$ $\longleftrightarrow$ $\omega_0 = \omega$

Note: for visible light & macroscopic times, ω needs to be very close to ω_0 for $|c_b|^2$ to be large re for transitions

Note: this result says $|c_b|^2$ will oscillate but we've only done first order perturbation theory so we have not really shown that yet

The result $|c_b|^2 \propto t^2$ for short times is unexpected → transition rates where $|c_b|^2 \propto t$ seem more normal. You'll see that under many circumstances $|c_b|^2 \propto t$ (essentially) does happen