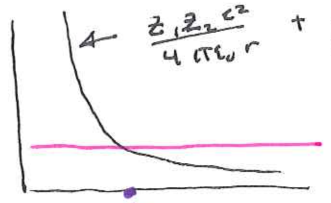
α decay $\quad {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + \alpha \rightarrow \begin{matrix} 2p \\ 2n \end{matrix}$



turning pt $b \gg$ $\overbrace{\text{nuclear size}}$

force range

$R \approx \begin{matrix} 1.4 \\ 1.1 \end{matrix} A^{1/3}$ fm $\rightarrow 10^{-15}$

charge radius

Coulomb energy @ $R \sim \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r} = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \hbar c} \frac{\hbar c}{r}$

$\frac{e^2}{4\pi\epsilon_0 \hbar c} = \alpha$

$\sim Z_1 Z_2 \alpha \frac{197\ \text{MeV}\cdot\text{fm}}{\text{few fm}} \approx Z_1 Z_2$ MeV

$Z_1 = 2$, $Z_2 \sim 90$

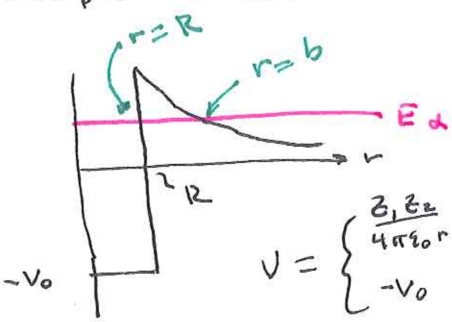
much larger than E_α!

$\hbar c$: "conversion constant"
197 MeV · fm
197 eV · nm

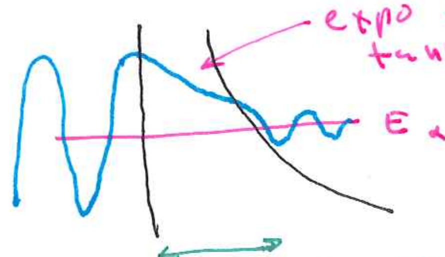
Centrifugal potential @ $R \sim \frac{\hbar^2(\ell+1/2)^2}{2mr^2} = \frac{\hbar^2 c^2 (\ell+1/2)^2}{2mc^2\ r^2}$

$\sim \left(\frac{197\ \text{MeV}\cdot\text{fm}}{r}\right)^2 \frac{(\ell+1/2)^2}{2\cdot 4000} \sim$ few MeV (not real small)

Simple model:



$V = \begin{cases} \frac{Z_1 Z_2}{4\pi\epsilon_0 r} & r > R \\ -V_0 & r < R \end{cases}$



classically disallowed

WKB $\psi \sim e^{-\int^x \kappa\, dx}$

$\kappa = \sqrt{\frac{2m}{\hbar}}(V-E)^{1/2}$

Barrier Tunneling Prob $= T$

$= e^{-2\int_R^b \kappa\, dx}$

estimates:

$R \sim 2-4$ fm

$E_\alpha \sim 2-8$ MeV

$V_0 \sim 50$ MeV

$Z \sim 50-100$

assume $l=0$

$$G = \sqrt{\frac{2m}{\hbar^2}}\left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r} - E_\alpha\right)^{1/2} = \sqrt{\frac{2mE}{\hbar^2}}\left(\frac{\overbrace{\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 E}}^{=b}}{r} - 1\right)^{1/2}$$

$$\int_R^b \sqrt{\frac{2mE}{\hbar^2}}\left(\frac{b}{r}-1\right)^{1/2} dr \underset{x = \frac{r}{b}}{=} \underbrace{\sqrt{\frac{2mE}{\hbar^2}}\, b}_{\sqrt{\frac{2m}{E}}\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar c} c \;=\; \sqrt{\frac{2mc^2}{E}} Z_1 Z_2 \alpha} \int_{R/b}^{1}\left(\frac{1}{x}-1\right)^{1/2} dx$$

$x = \sin^2\theta \quad \Big|^{\pi/2}_{\sin^{-1}(\sqrt{R/b}) = \theta_1}$

$$\int_{R/b}^{1}\left(\frac{1}{x}-1\right)^{1/2} dx = \int_{\theta_1}^{\pi/2}\left(\frac{\cos^2\theta}{\sin^2\theta}\right)^{1/2} 2\sin\theta\cos\theta$$

$$= \int_{\theta_1}^{\pi/2} 2\cos^2\theta\, d\theta = \int_{\theta_1}^{\pi/2}(\cos(2\theta)+1)\, d\theta$$

$$= \underbrace{\frac{1}{2}\sin(2\theta)}_{\sin\theta\cos\theta\,\Big|^{\pi/2}_{\theta_1}} + \frac{\pi}{2} - \theta_1$$

$$\sin\theta\cos\theta\,\Big|^{\pi/2}_{\theta_1} = -\sin\theta_1\cos\theta_1 = -\sqrt{\frac{R}{b}}\Big(1-\underbrace{\frac{R}{b}}_{\sin^2\theta_1}\Big)^{1/2}$$

$$G = \sqrt{\frac{2mc^2}{E}} Z_1 Z_2 \alpha \left[\frac{\pi}{2} - \sin^{-1}\left(\sqrt{\frac{R}{b}}\right) - \sqrt{\frac{R}{b}}\left(1-\frac{R}{b}\right)^{1/2}\right]$$

$$b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar c}\frac{\hbar c}{E} = \underset{2\;\;100}{Z_1 Z_2}\alpha \frac{197\text{ MeV}\cdot\text{fm}}{E \leftarrow 4\text{ MeV}} = 72\text{ fm}$$

$\sqrt{\frac{3\text{ fm}}{72\text{ fm}}} = .2 \leftarrow$ smallish

$$G \approx \sqrt{\frac{2mc^2}{E}} Z_1 Z_2 \alpha \frac{\pi}{2}$$

$$T = e^{-2G}$$

$$\text{decay rate} = \frac{\text{prob exit}}{s} = T\left(\frac{\text{incoming prob}}{s}\right)$$

crude estimates: $|\psi|^2$ velocity

$\frac{1}{\frac{4}{3}\pi R^3}$

1-d: $\frac{\text{velocity}}{2R} \sim \frac{1}{\tau_0}$

$$\tau_{1/2} \sim G^{2G}\, \tau_0$$

Eg α decay

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + \alpha$$

50770   43583   2603 $\leftarrow$ (mass $-$ A) $\times 10^6$

$\underbrace{43583 \quad 2603}_{46186}$ downhill

$\Delta m c^2 = 4.27$ MeV

$931.5 \frac{\text{MeV}}{u}$

p decay?

$${}^{238}_{92}U \rightarrow {}^{237}_{91}Pa + p$$

50770   51080   7825

$\underbrace{51080 \quad 7825}_{58905}$ uphill

n decay

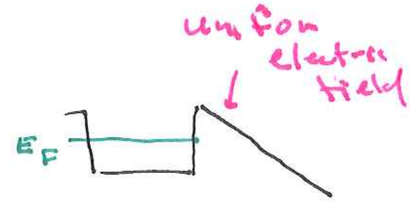
$${}^{238}_{92}U \rightarrow {}^{237}_{92}U + n$$

50770   48608   8665

$\underbrace{48608 \quad 8665}_{57273}$ uphill

other applications—

if $kT \ll W$ "cold cathode emission"



scanning tunneling Microscopy

tunneling thru oxide layer