Zeeman in Hydrogen L_α : $-\frac{q}{2m}\vec{B}\cdot\vec{L}$ $-\mu\cdot B$

$\frac{q}{2m} g \vec{S}$ → 2

seek $\langle S_z\rangle$ ← in terms of m_j

$-\frac{q}{2m}(\overrightarrow{L+2S})\cdot\vec{B}$

$J+S$

$\hbar m_j B$ $\quad -\frac{qB}{2m}\langle S\rangle$

→ $2P_{3/2}$, → $2P_{1/2}$ } $\overrightarrow{1+\frac{1}{2}} = \frac{1}{2}\ \frac{3}{2}$

→ $S_{\frac{1}{2}}$

$\frac{3}{2}\ \frac{3}{2}$ $\quad 1\uparrow$

$\frac{3}{2}\ \frac{1}{2}$ $\quad \sqrt{\frac{1}{3}}\,1\downarrow + \sqrt{\frac{2}{3}}\,0\uparrow$ $\qquad \frac{1}{2}\ \frac{1}{2}$ $\quad \sqrt{\frac{2}{3}}\,1\downarrow - \sqrt{\frac{1}{3}}\,0\uparrow$

$\frac{3}{2}\ -\frac{1}{2}$ $\quad \sqrt{\frac{2}{3}}\,0\downarrow + \sqrt{\frac{1}{3}}\,-1\uparrow$ $\qquad \frac{1}{2}\ -\frac{1}{2}$ $\quad \sqrt{\frac{1}{3}}\,0\downarrow - \sqrt{\frac{2}{3}}\,-1\uparrow$

$\frac{3}{2}\ -\frac{3}{2}$ $\quad -1\downarrow$

$$\left.\begin{aligned}
\langle \tfrac{3}{2}\tfrac{3}{2}|S_z|\tfrac{3}{2}\tfrac{3}{2}\rangle &= \tfrac{1}{2} = \tfrac{1}{3}m_j\\
\langle \tfrac{3}{2}\tfrac{1}{2}|S_z|\tfrac{3}{2}\tfrac{1}{2}\rangle &= \tfrac{1}{3}(-\tfrac{1}{2}) + \tfrac{2}{3}(\tfrac{1}{2}) = \tfrac{1}{6} = \tfrac{1}{3}m_j\\
\langle \tfrac{3}{2}-\tfrac{1}{2}|S_z|\tfrac{3}{2}-\tfrac{1}{2}\rangle &= \tfrac{2}{3}(-\tfrac{1}{2}) + \tfrac{1}{3}\tfrac{1}{2} = -\tfrac{1}{6} = \tfrac{1}{3}m_j\\
\langle \tfrac{3}{2}-\tfrac{3}{2}|S_z|\tfrac{3}{2}-\tfrac{3}{2}\rangle &= -\tfrac{1}{2} = \tfrac{1}{3}m_j
\end{aligned}\right\} -\frac{q\hbar B m_j}{2m}\left(1+\frac{1}{3}\right)$$

$$\left.\begin{aligned}
\langle \tfrac{1}{2}\tfrac{1}{2}|S_z|\tfrac{1}{2}\tfrac{1}{2}\rangle &= \tfrac{2}{3}(-\tfrac{1}{2}) + \tfrac{1}{3}(\tfrac{1}{2}) = -\tfrac{1}{6} = -\tfrac{1}{3}m_j\\
\langle \tfrac{1}{2}-\tfrac{1}{2}|S_z|\tfrac{1}{2}-\tfrac{1}{2}\rangle &= \tfrac{1}{3}\cdot\tfrac{-1}{2} + \tfrac{2}{3}\tfrac{1}{2} = +\tfrac{1}{6} = -\tfrac{1}{3}m_j
\end{aligned}\right\} -\frac{q\hbar B m_j}{2m}\left(1-\frac{1}{3}\right)$$

$-\frac{q\hbar B m_j}{2m}(1+1)$

gs: $\langle S_z\rangle = m_j$

selection rules: $\Delta m = \pm 1, 0$ → if ( ) were same 3 lines

alt approach: $\vec{S} \sim \frac{(S\cdot J)\vec{J}}{J^2}$

Wigner-Eckart $\langle j m|T^k_q|j'm'\rangle = \underbrace{\langle j\|T^k\|j'\rangle}_{\text{does not depend on } m_s}\overbrace{\langle j'm';kq|jm\rangle}^{\text{C-G}}$

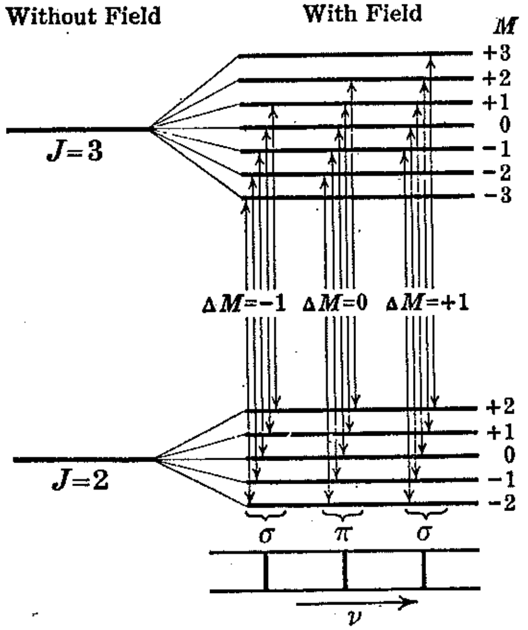

Fig. 44. Normal Zeeman Effect for a Combination $J = 3 \rightarrow J = 2$. The arrows representing the transitions form three groups (indicated by brackets). The arrows in each group have equal length and give rise, therefore, to one and the same line in the splitting pattern (lower part of figure).

This kind of splitting is called the *normal Zeeman effect.* It is observed *only for singlet lines* ($S = 0$). [Cf. Fig. 39(*a*), p. 97.]

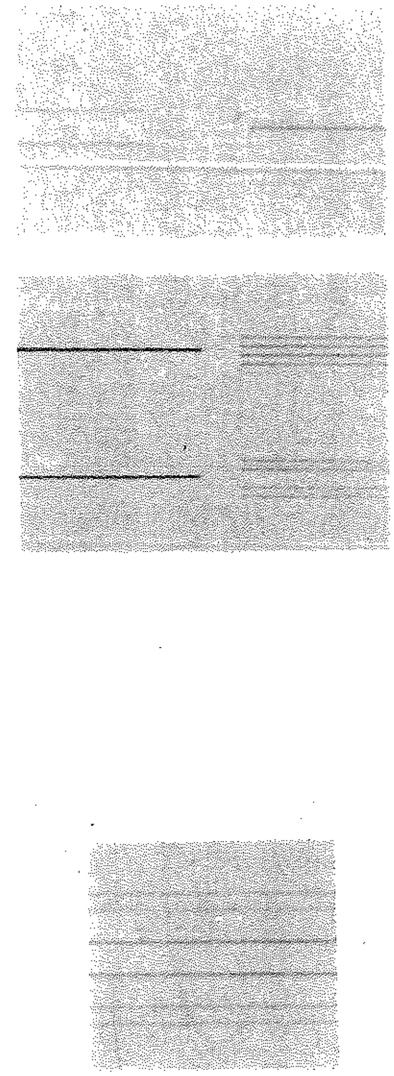

Fig. 39. Examples of Line Splitting in a Magnetic Field (Zeeman Effect) [after Back and Landé (6)].
(*a*) *Normal* Zeeman triplet of the Cd line 6438.47 Å ($^1P - {}^1D$ transition). Above, the exposure was so made that only light polarized parallel to the field direction could reach the plate (single component at the position of the original line). Below, the components were polarized perpendicular to the field; they lie symmetrical to the original line.
(*b*) *Anomalous* Zeeman splitting of the two D lines of Na, 5889.96 Å and 5895.93 Å ($^2S - {}^2P$ transition). Above, with magnetic field. Below, without magnetic field.
(*c*) *Anomalous* splitting of the Zn line 4722.16 Å ($^3P_1 - {}^3S_1$ transition).

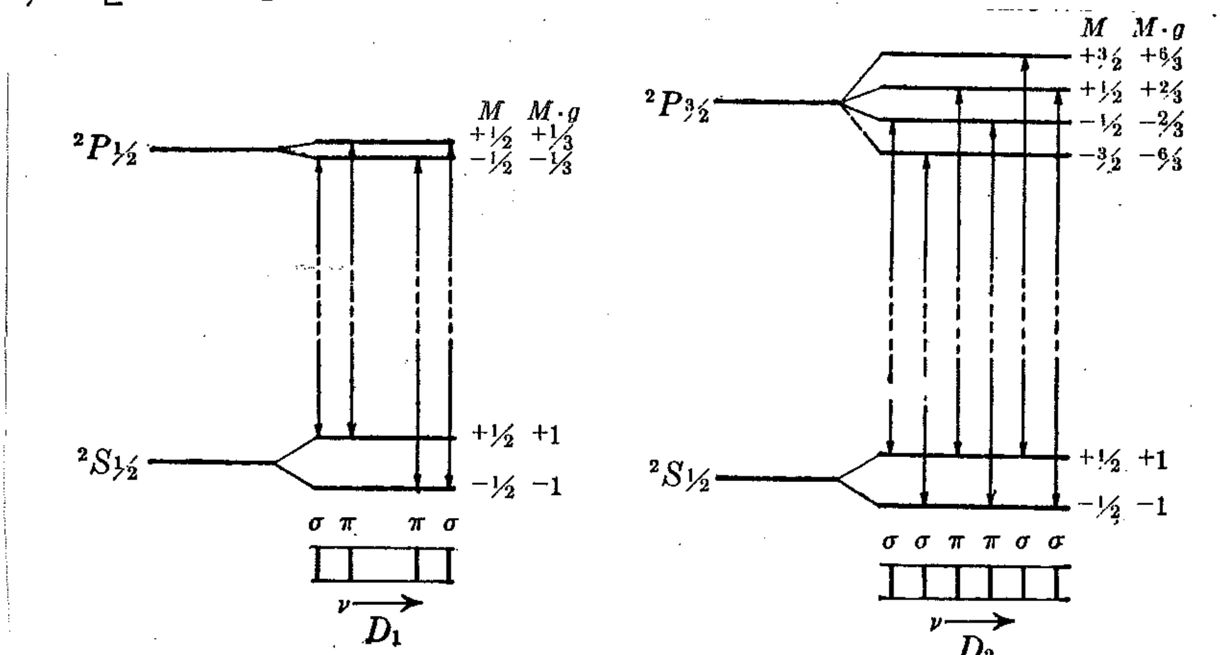

Fig. 45. Anomalous Zeeman Effect of the Sodium D Lines, $^2P_{1/2} \rightarrow {}^2S_{1/2}$ and $^2P_{3/2} \rightarrow {}^2S_{1/2}$. [Cf. Fig. 39 (*b*), p. 97.] The components designated by σ have $\Delta M = \pm 1$; those designated by π have $\Delta M = 0$. It should be noted that, contrary to Fig. 44, arrows indicating transitions with equal ΔM no longer have the same length, because of the difference in the splitting in the upper and lower states.