Example 3 in chapter 2: free particle [ie no forces; $V=0$]

TDSE: $-\frac{\hbar^2}{2m}\partial_x^2 \Psi = i\hbar \partial_t \Psi$; separation $\Rightarrow \Psi = \psi(x)\, e^{-i\frac{Et}{\hbar}}$

TISE: $-\frac{\hbar^2}{2m}\partial_x^2 \psi = E\psi \rightarrow \partial_x^2\psi = \underbrace{\frac{-2mE}{\hbar^2}}_{\text{define as } k^2}\psi$ define $\frac{E}{\hbar} = \omega$

solutions: $\underbrace{e^{ikx}, e^{-ikx}}$, $\sin(kx)$, $\cos(kx)$

we select these & interpret as waves moving:

$e^{ikx} \rightarrow$; $e^{-ikx} \leftarrow$

and we can put these two together simply by allowing $k \in (-\infty, \infty)$

So: $\Psi = e^{i(kx - \omega t)}$ where $\omega = \frac{\hbar k^2}{2m}$

phase velocity $= \frac{\omega}{k}$; group velocity $= \frac{d\omega}{dk} = \frac{\overbrace{\hbar k}^{p}}{m} = \underbrace{\text{classical velocity}}$

$= \frac{1}{2}$ classical velocity ← who cares?

we needed this in order for QM to look like Newton

Solve initial value problem with Fourier Transform

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(k)\, e^{ikx}\, dk$$ ← a sum over different wavelengths given by $\lambda = \frac{2\pi}{k}$ with amplitudes $g(k)$

$$g(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$ ← we can find amplitudes from $f(x)$

see the first eq as a sum over states like $\sum c_n \psi_n$

see the second eq as like $\langle e^{ikx} | F \rangle = c_n$

to find the future of a state we tacked on $e^{-i\omega t}$ as in $\sum c_n \psi_n \rightarrow \sum c_n \psi_n e^{-i\omega_n t}$, same goes here

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(k)\, e^{ikx}\, e^{-i\overbrace{\frac{\hbar k^2}{2m}}^{\omega} t}\, dk$$

Note: for these free particle states any (positive) value of energy is allowed → "continuous" energy

For problems with bounded classically allowed region (ie 2 turning pts with standing waves made of $\leftrightarrows$) we had energy labeled with whole numbers "discrete" energy