Complete no more than 5 of the following problems

1. (R-R) Consider the problem of two attractive delta function potentials symmetrically located about the origin at $x = \pm a$, with dimensionless Hamiltonian:

$$H = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x)$$

where

$$V(x) = -\left[\delta(x-a) + \delta(x+a)\right]$$

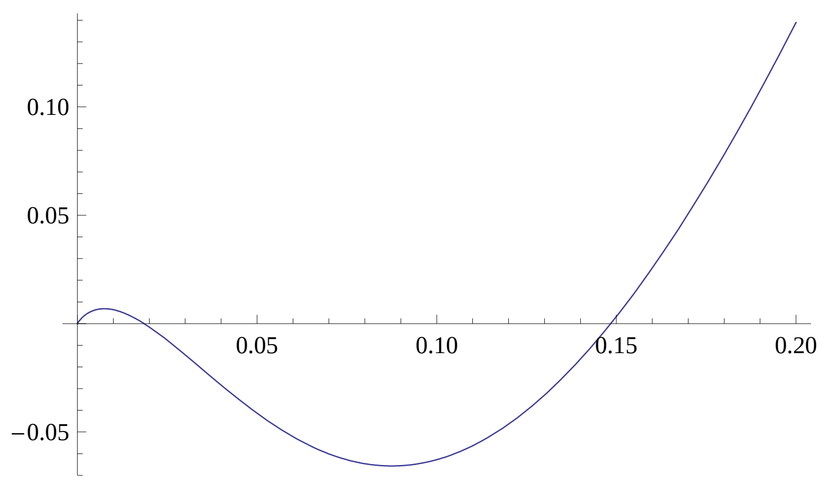
The trial function $f(x) = xe^{-bx^2}$ is used to evaluate $\langle H\rangle = \langle f|H|f\rangle/\langle f|f\rangle$:

(a) Calculate: $\langle f|V(x)|f\rangle$

(b) Calculate $\langle f|f\rangle$ (You may find the integrals on the cheat sheet helpful.)

(c) The below plot shows $\langle H\rangle$ plotted as a function of $b \in [0, .2]$ for $a = 2$. Use this graph to make as precise statement as you can about the exact eigenenergies of this system.

(d) Suggest a trial wavefunction to approximate the ground state of this system.



2. (R-R) The ground state energy of a system is estimated both by the Rayleigh-Ritz method and by a second-order perturbation theory calculation. The Rayleigh-Ritz result is -27.1 eV; the perturbation theory result is -26.0 eV. Which lies closer to the true ground state energy? Why? What is required for the R-R method to give an accurate bound for an excited state?

3. (WKB+) Consider the potential

$$V(x) = \lambda\,|x|$$

where λ is a positive constant. Provide rough sketches the three lowest energy eigenfunctions. Label the ground state ψ_1, the first excited state ψ_2, etc. Consider the symmetry of these three states: record which state(s) are even, odd, or of no definite symmetry. Some integrals must zero simply because of symmetry. Which (if any) of the following integrals are zero because of symmetry: $\langle\psi_2|\psi_2\rangle$, $\langle\psi_1|\psi_2\rangle$, $\langle\psi_3|x|\psi_1\rangle$, $\langle\psi_3|x|\psi_2\rangle$, and $\langle\psi_1|\frac{d}{dx}|\psi_2\rangle$. Sketch a large n wavefunction (ψ_n), being careful to show wavefunction behavior (x variation of amplitude and wavelength) as required by WKB.

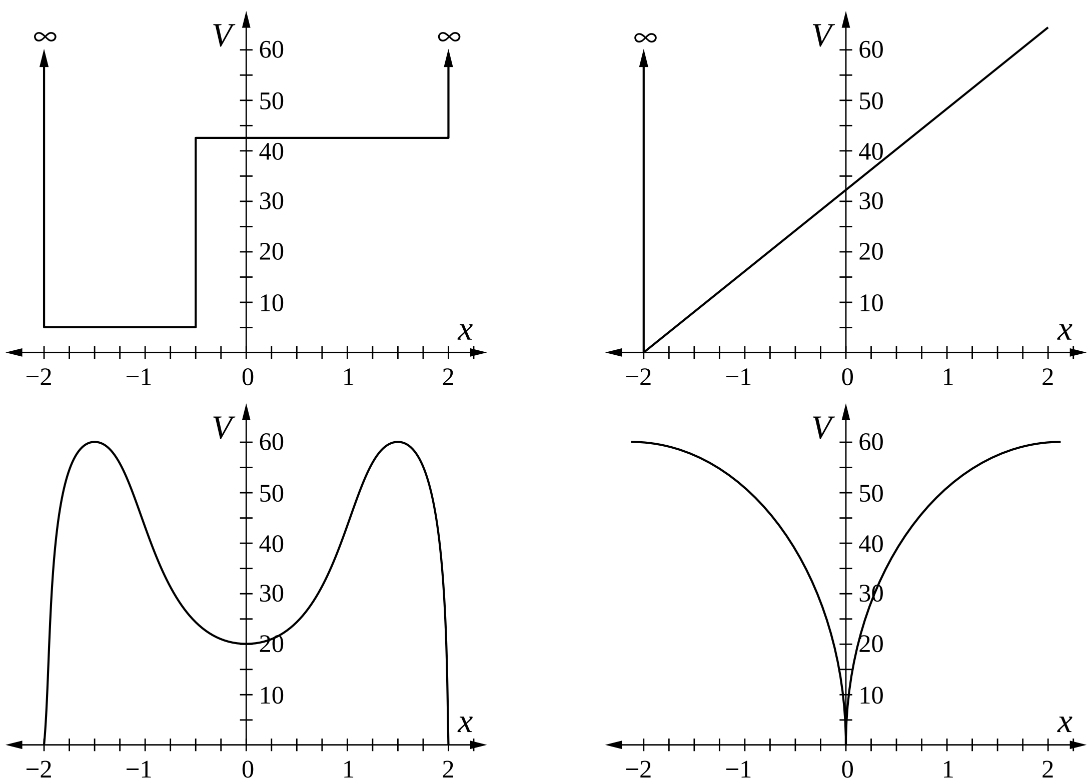
4. (WKB) Kirkman writes the WKB integral in the form:

$$\int_a^b k(x)dx = \pi(n - \text{something}) \qquad \text{where: } k(x) = \frac{\sqrt{2m(E - V(x))}}{\hbar} \tag{1}$$

(a) For each of the below four plots of $V(x)$ report the values for a, b, and "something" if we are considering bound state (or quasi bound state) wavefunctions ψ with an energy, E, of 50.

(b) The lower left potential for $E = 50$ has the "quasi bound state" mentioned above. How does this quasi bound state differ from the other states which are truly bound states?

(c) For each of the below potentials, assume that the integral of Eq. 1 produces $n = 6$ for $E = 50$. Sketch the corresponding WKB wavefunction ψ directly on each of the below plots properly displaying changing wavelength & amplitude and behavior near a & b.



5. (TDPT) We derived:

$$c_b(t) \approx -\frac{i}{\hbar} \int_0^t H'_{ba}(t')\, e^{i(E_b - E_a)t'/\hbar}\, dt'$$

I now ask you exactly what this beast is about. I.e., what do the symbols: $c_b(t)$, $H'_{ba}(t')$, E_a and E_b mean? What are the assumptions behind this equation (e.g., why is the integral from 0 to t)? The next step in TDPT reports:

$$P_{a \to b} \approx \frac{|V_{ab}|^2}{\hbar^2} \frac{\sin^2\left((\omega_0 - \omega)t/2\right)}{(\omega_0 - \omega)^2}$$

You've got some additional symbols to define: $P_{a \to b}, \omega_0, \omega, V_{ab}$. Again what are the assumptions behind this equation (e.g., what happened to H')?

6. (TIPT) Consider the perturbation λx^2 added to the harmonic oscillator Hamiltonian. Find the first and second order corrections to the energy of the ground state. (Recall: a_+, a_-)

7. (dTIPT) Consider a particle-in-a-2d-box with $V(x,y) = 0$ for $0 < x < L$ and $0 < y < L$ and $V(x,y) = \infty$ elsewhere. The eigenenergies depend on two whole-number quantum numbers: n_x and n_y:

$$E = \frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2 + n_y^2\right)$$

with corresponding normalized eigenfunction:

$$\psi_{n_x n_y}(x,y) = \frac{2}{L}\,\sin\left(n_x \pi x/L\right)\sin\left(n_y \pi y/L\right)$$

A perturbing 2d delta function potential is placed at the location $(x,y) = (L/3, L/4)$:

$$V' = \lambda\;\delta(x - L/3)\;\delta(y - L/4)$$

Consider the first-order energy shift of the degenerate ψ_{12} and ψ_{21} states. Find the matrix representing V' in this degenerate sub-space. Find the matrix's eigenvalues and vectors. Report the "good" wavefunctions and the corresponding approximate eigenenergy (accurate to first-order) for those "good" wavefunctions. FYI: $\sin(\pi/3) = \sin(2\pi/3) = \sqrt{3}/2$, $\sin(\pi/4) = 1/\sqrt{2}$, $\sin(\pi/2) = 1$