

1) a) f is amplitude of scattered wave as in $f \frac{e^{ikr}}{r}$. As a result

$$|f|^2 = \frac{d\sigma}{d\Omega}$$

For large r the exact solution to SE will be $\frac{1}{r} \cos(kr - \frac{\pi}{2}(l+1) + \delta_l)$
 whereas in the absence of a potential $\sim \frac{1}{r} \cos(kr - \frac{\pi}{2}(l+1) + 0)$
 So δ_l is a phase shift caused by potential

3) For small k , eg $k=0.5$, the plots show $\delta_0, \delta_2 \sim \pi$
 ie only δ_0 is significant ($\sin \delta \neq 0$) $\delta_l = 0$ for $l > 2$
 so \sum_l in formula for f reduces to $l=0$ term
 alone so $f \sim P_0(\cos \theta)$ which is a constant

At $k=1.4656$ both $\delta_0 \neq \delta_3$ are significant

so $f \sim P_0 + P_3 \rightarrow |P_3|^2$ by itself has

zeros at $\cos \theta = 0, \pm 0.77$ ie $\theta = .69, \pi/2, 2.5$

which is a good match $\uparrow \sqrt{\frac{3}{5}}$

Note that $\sin(\delta_3)$ is zero both for k a bit less than
 1.4656 and for k greater than 1.4656 — so
 there will be an extra contribution to σ
 just at $k=1.4656$ — ie a resonant peak