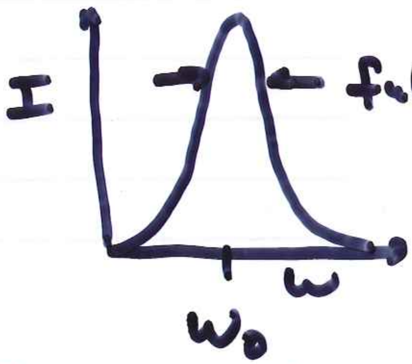
$$\text{Power} = \tfrac{1}{2} V_0 I_0 \cos\phi = V_{rms} I_{rms} \cos\phi$$

"power factor"

angle between V & I

Note: you are paying for more than you get unless $\phi = 0$ - beware inductors/motors

Resonance



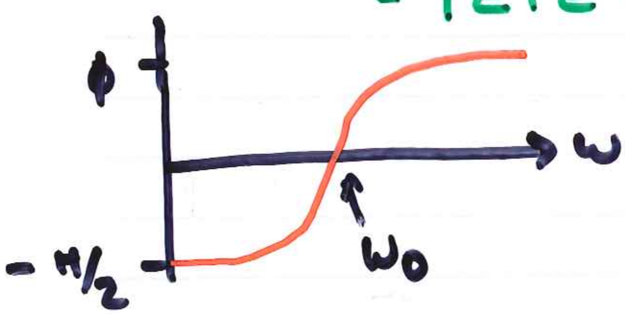
full width at $\frac{1}{\sqrt{2}}$ max

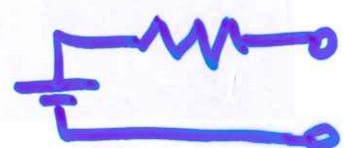
$$= \frac{\omega_0}{Q} \quad ; \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

series LRC : $Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$

$$= |Z| e^{i\phi}$$

$$I = \frac{V_0}{\underbrace{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}_{|Z|}}$$





Thevenin : Any two-wire output of complex circuit looks like ...

$f = f_0 e^{i\omega t}$

$g = g_0 e^{i\omega t}$

$$\overline{\mathrm{Re}(f)\,\mathrm{Re}(g)} = \tfrac{1}{2}\mathrm{Re}(f^* g)$$

Impedance match ($Z_{in} = Z^*_{out}$) for max power transfer (but often independence more desired than power transfer)

$\Rightarrow Z_{in} \gg Z_{out}$

"displacement current"

$$\nabla\cdot D = \rho$$

$$\nabla\times E = -\partial_t B$$

$$\nabla\cdot B = 0$$

$$\nabla\times H = J \; + ? \leftarrow \partial_t D$$

$$\nabla\cdot J + \partial_t \rho = 0 \leftarrow$$

Continuity

$$\nabla\cdot(\nabla\times H) = 0$$

$$\begin{vmatrix} \partial_x & \partial_y & \partial_z \\ \partial_x & \partial_y & \partial_z \\ - & H & - \end{vmatrix} = 0$$

$$E = \epsilon D$$

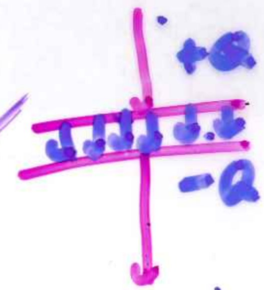
$$B = \mu H$$

μ_0

$$\partial_t \rho \leftarrow \nabla\cdot D$$

$$\nabla\cdot J + \nabla\cdot ? = 0$$

$$+\,\underbrace{\nabla\cdot\partial_t D}_{\rho}$$

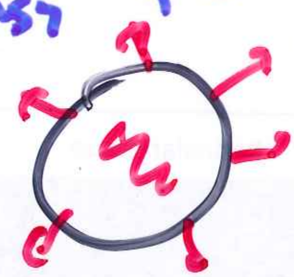


→ "Scott" →

$$u_M = \frac{1}{2} B\cdot H \leftarrow \frac{\text{Ener}}{\text{Volu}}$$

$$u_E = \frac{1}{2} E\cdot D$$

Conservation : Energy flow ←



$$\partial_t\left(\frac{\text{Energy}}{\text{Vol}}\right) = -\nabla\cdot\vec{S}$$

Heaviside ← EE

Poynting

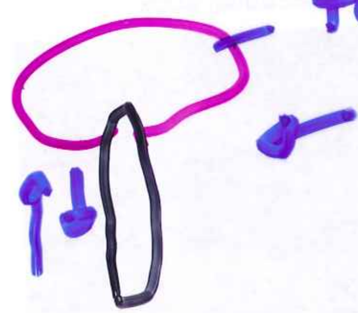
$$\nabla\cdot(E\times H) = \underbrace{\nabla\times E}_{-\partial_t B}\cdot H - E\cdot\underbrace{\nabla\times H}_{J+\partial_t D}$$

$$= -\Big(\underbrace{E\cdot\partial_t D}_{\frac{1}{2}\partial_t E\cdot D} + \underbrace{\partial_t B\cdot H}_{\frac{1}{2}\partial_t B\cdot H}\Big) - E\cdot J$$

$$= -\partial_t(u_E + u_M) - E\cdot J$$

$$\nabla\cdot(\underbrace{E\times H}_{S}) + \partial_t(u_E + u_M) = \overbrace{-E\cdot J}$$

$+Q$



Joul Heating

l $\quad E = \dfrac{V}{l}$ $\qquad \dfrac{I}{A} = J$

$$E\cdot J = \frac{VI}{lA} = \frac{\text{heat}}{\text{volume}}$$

Waves? $B=\mu H$ $D=\varepsilon E$ $J=g E$

$$\nabla\times H = J + \partial_t D$$

$$\nabla\times(\nabla\times H) = \nabla(\underbrace{\nabla\cdot H}_{0}) - \nabla^2 \vec{H}$$

$$= \nabla\times J + \partial_t \underbrace{\nabla\times D}_{\varepsilon\nabla\times E}$$

$\nabla\times J = g\,\nabla\times E$

Wave Eq!!!

$$\left(\partial_x^2 - \frac{1}{v^2}\partial_t^2\right)P = 0$$

$$P = f(x-vt) + g(x+vt)$$

$\partial_x^2 \to \nabla^2$

$$= (g + \varepsilon\partial_t)\underbrace{\nabla\times E}_{-\partial_t B}$$

$$= (g + \varepsilon\partial_t)(-\partial_t B)$$

$B = \mu H$

$$= -g\mu\,\partial_t H - \varepsilon\mu\,\partial_t^2 H$$

$$0 = \underbrace{\left(\nabla^2 - \varepsilon\mu\,\partial_t^2\right)}_{\text{wave eq}} H - g\mu\,\partial_t H$$

$$\frac{1}{v^2} = \varepsilon\mu \,;\quad v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\kappa\kappa}} \qquad c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$v^2 = \frac{1}{\varepsilon\mu} \Rightarrow v = \frac{1}{\sqrt{K K_M}} c$$

$$\frac{c}{v} \; \cancel{\frac{v}{c}} = n \leftarrow 1.33 \text{ wat}$$

$$n = \sqrt{KK}$$

$$K = 81$$

$$1.33 = \sqrt{81 \cdot 1} = 9$$

$$1.33 = \frac{\cancel{\Delta}}{?}$$

$$\nabla \times \left(\nabla \times E \right) = -\cancel{\partial_t} \left(B \right) \longrightarrow \nabla \times B = \mu J + \mu \partial_t \overset{\varepsilon E}{D}$$

$$\mu H$$

$$\nabla \times H = J + \partial_t D$$

$$\nabla \left(\underbrace{\nabla \cdot E} \right) - \nabla^2 E$$

$$\frac{1}{\varepsilon} \nabla \cdot D = \frac{1}{\varepsilon} \rho$$

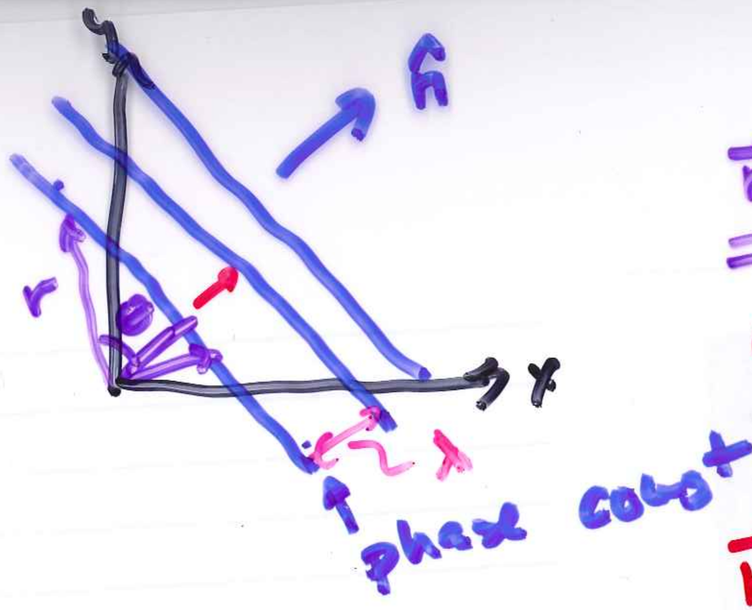
$$\nabla \frac{1}{\varepsilon} \rho - \nabla^2 E = -\mu \varepsilon \partial_t^2 E - \partial_t \mu J$$

$$J = gE$$

$$\nabla^2 - \frac{1}{v^2} \partial_t^2$$

$$\overbrace{\nabla \frac{1}{\varepsilon} \rho}^{0} = \left(\nabla^2 - \mu \varepsilon \partial_t^2 \right) E - \mu g \partial_t E$$

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$



$$\vec{r}\cdot\hat{n} = \text{const}$$

$$\Delta r = 2\pi\lambda$$

$$\vec{k} = \frac{2\pi}{\lambda}\hat{n}$$

$$e^{i(\vec{k}\cdot\vec{r})}$$

$$e^{i(\vec{k}\cdot\vec{r}-\omega t)} \checkmark$$

$$\vec{r}\cdot\vec{k}$$

$$e^{-i(\omega t-\vec{k}\cdot\vec{r})} \checkmark$$

$$f(\underbrace{x - vt}), \quad v = f\lambda$$

$$\frac{1}{\lambda}x - ft$$

$$kx - \omega t$$

$$\partial_x e^{i(\vec{k}\cdot\vec{r}-\omega t)} = ik_x(\ \ )$$

$$\vec{\nabla} e^{i(\vec{k}\cdot\vec{r}-\omega t)} = i\vec{k}(\ \ )$$

$$\partial_t e^{i(\vec{k}\cdot\vec{r}-\omega t)} = -i\omega(\ \ )$$

$$\boxed{\begin{aligned}\vec{\nabla} &\to i\vec{k}\\ \partial_t &\to -i\omega\end{aligned}}$$

$$\nabla \times E = -\partial_t B = -\mu \partial_t H$$

$$\vec{H} = \vec{H}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\vec{E} = \vec{E}_0 \quad ..$$

$$i\vec{k} \times \vec{E}_0 = i\omega\mu \vec{H}_0$$

$$(\nabla^2 - \epsilon\mu\partial_t^2) \begin{matrix} H \\ E \end{matrix} = 0$$

$$(\underbrace{-k^2 + \epsilon\mu\omega^2}) \begin{matrix} H_0 \\ E_0 \end{matrix} = 0$$

$$k = \sqrt{\epsilon\mu}\,\omega$$

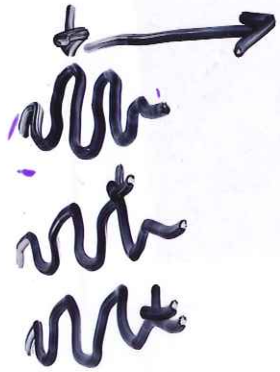
$$v = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

crest velocity

phase velocity

?? group velocity

$$v_g = \frac{d\omega}{dk}$$



$n = 1.33$

$n < 1$

$v = \frac{c}{n}$