Forces & Torques $\vec{F} = q\vec{v}\times B$ $\quad \vec{F} = q\vec{E}$

$I\,d\vec{l},\ \vec{K}\,da,\ \vec{J}\,dV$ $\quad \rho\,dV,\ \sigma\,da,\ \lambda\,dl$

magnetic dipole $\vec{m}$ $\vec{m}$

" Area · Current

Electric Dipole

$\vec{p}$ + −

$\vec{F} = 0$ if $\vec{B}$ uniform $\quad \vec{F} = 0$ if $\vec{E}$ uniform

$\vec{\tau} = \overrightarrow{m\times B}$ $\quad$ Factoid $\quad \vec{\tau} = \overrightarrow{p\times E}$

$PE = -\vec{m}\cdot\vec{B}$ $\quad \oint r_i\,dr_j = \begin{cases} 0 & i=j \\ \pm \text{Area} \end{cases}$ $\quad PE = -\vec{p}\cdot\vec{E}$

$\vec{F} = -\vec{\nabla} PE$ $\quad \oint r_i\,dr_j = -\oint r_j\,dr_i$ $\quad \vec{F} = -\nabla PE$

Method: index manipulation

Einstein summation notation

$(\vec{A}\times\vec{B})_i = \epsilon_{ijk} A_j B_k \leftarrow \sum_j \sum_k$ implied

$\begin{matrix} 1\,2\,3 \\ 1\,3\,2 \end{matrix}\Big\} \Rightarrow (A\times B)_x = A_y B_z - A_z B_y$

Levi-Civita $\rightarrow$ $\epsilon_{ijk} = \begin{cases} 0 & \text{unless } ijk \text{ distinct} \\ 1 & \text{if } ijk = 123 \text{ or cycle} \\ -1 & \text{otherwise} \end{cases}$

$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(A\cdot C) - \vec{C}(A\cdot B)$

$\epsilon_{ijk} A_j \epsilon_{kab} B_a C_b = (\delta_{ia}\delta_{jb} - \delta_{ib}\delta_{ja}) A_j B_a C_b$

$\epsilon_{ijk}\epsilon_{abk} = \delta_{ia}\delta_{jb} - \delta_{ib}\delta_{ja}$

"rotationally invariant tensors"

(1) B by integration

(2) Ampere's Law

(3) Laplace

B3.pdf

B2

$A \to B$

?

?

$\bar{m} \propto \bar{B}$

usually neglect

$\bar{m} - \bar{B}$

small diamagnetic

$\bar{m}$ paramagnetic

Liquid O_2

Ferromagnetic $\to$ Tc

$K_m = 1.0001$

$K \sim 1000$, $1{,}000{,}000$

$D = \epsilon_0 E + P$

$\nabla \cdot D = \rho_f$

$\frac{\sum \bar{p}}{\text{Volume}}$

$\bar{P} \propto \bar{E}$

$K \sim 2-3$

80

$\Phi = \frac{1}{4\pi\epsilon_0} \frac{\bar{p} \cdot (\vec{r} - \vec{r}')}{|r - r'|}$

$\bar{P}\, dV = \bar{p}$

$\int$

$\vdots$

$\rho_b = -\nabla \cdot P$

$J_b = P \cdot n$

$$\vec{A} = \frac{\mu}{4\pi}\,\frac{\vec{m}\times(r-r')}{|r-r'|^3}$$

$$\vec{M}\,dV = \vec{m}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{M}\,dV\times(r-r')}{|r-r'|^3}$$

$$-\nabla\frac{1}{|r-r'|}$$

$$\nabla'$$

$$= \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{|r-r'|}$$

$$= \frac{\mu_0}{4\pi}\int \vec{M}(r')\times\nabla'\frac{1}{|r-r'|}\,dV$$

$$\vec{F}\times\nabla\phi$$

$$\nabla\times(\phi F) = \phi\,\nabla\times F - F\times\nabla\phi$$

$$= \frac{\mu_0}{4\pi}\int \frac{1}{|r-r'|}\nabla'\times M - \int \nabla'\times\left(\frac{M}{|r-r'|}\right)$$

$$\int \nabla\times F\,dV = \int \hat{n}\times\vec{F}\,dA$$

$$= \frac{\mu_0}{4\pi}\int \frac{\nabla'\times M}{|r-r'|}\,dV$$

$$J_B = \nabla\times M$$

$$= \frac{\mu_0}{4\pi} \int \frac{J_b}{|r-r'|} + \oint \frac{M \times \hat{n}}{|r-r'|} \, dA$$

$\nabla \times M$

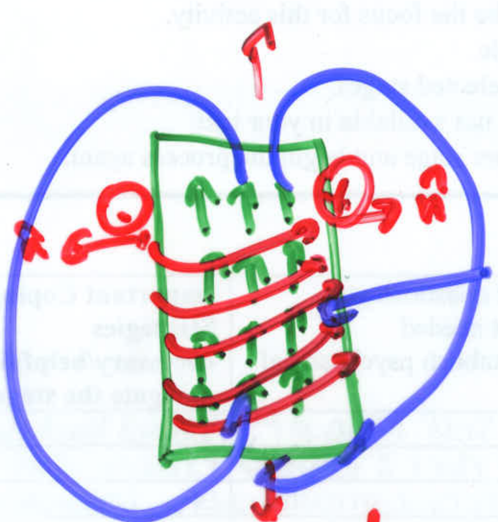
$\vec{K} = \vec{M} \times \hat{n}$

$M(\vec{r}\,') \longrightarrow \nabla \times M$ ✓

$M \times n$

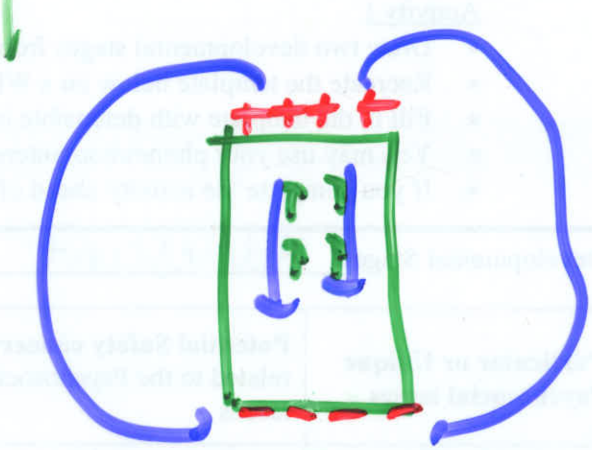
$P \longleftrightarrow -\nabla \cdot P$

$P \cdot n$



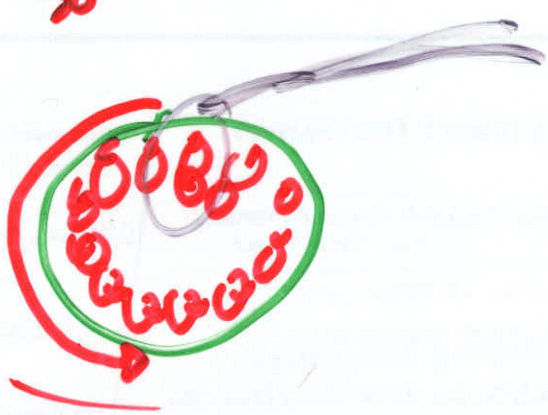
$B = \mu_0 K = \mu_0 M$

Solenoid



$\rho = -\nabla \cdot P$

$\nabla \cdot E = \frac{1}{\varepsilon_0}(\rho_f + \rho_b)$



$\nabla \cdot \varepsilon_0 E = \rho_f - \nabla \cdot P$

$\nabla(\underbrace{\varepsilon_0 E + P}_{D}) = \rho_f$

$$\nabla \times B = \mu_0 (J)$$

$$J_f + \nabla \times M$$

$$\nabla \times \frac{1}{\mu_0} B = J_f + \nabla \times M$$

$$P = \epsilon_0 \chi E$$

$$D = \epsilon_0 (1+\chi) E$$

$$\nabla \times \underbrace{\left(\frac{1}{\mu_0} B - M \right)}_{H} = J_f$$

$$M = \chi H$$

$$\frac{1}{\mu_0} B - M = H$$

$$\frac{1}{\mu_0} B = (1+\chi) H$$

$$B = \underbrace{\mu_0 (1+\chi)}_{\mu} H$$

$$K = 1 + \chi$$

$$1 \pm .0001$$

$$100{,}000$$

PM: M

No free currents

$$\nabla \times H = 0$$

$$H = -\nabla \phi$$

$$\nabla \cdot H =$$

$$\nabla \times E = 0$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho}{|r - r'|} dv$$

$$\nabla \cdot \left(\underset{0}{\frac{1}{\mu_0} B} - M \right) = \underbrace{-\nabla \cdot M}_{\rho_{pole}}$$

$H = -\nabla \phi$

$\leftarrow -\nabla \cdot M$

$\phi = \frac{1}{4\pi} \int \frac{\rho_{pol}}{|r - r'|} \, dV$

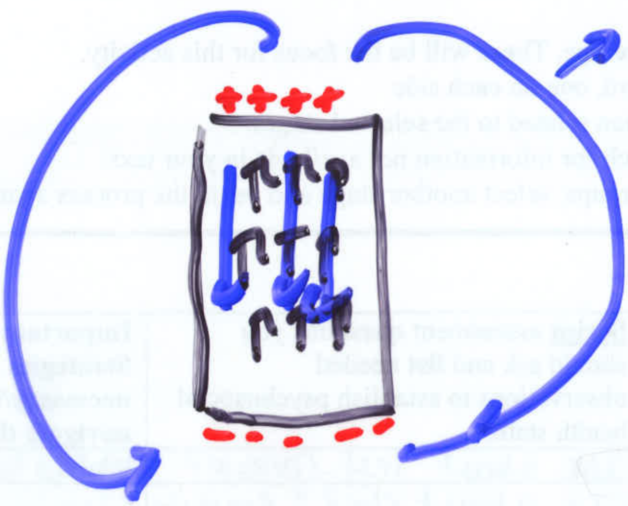
$M \cdot n$

"σ"

$\rho \, dV \rightarrow -\nabla \cdot M \, dV$

$\sigma \, dA \rightarrow M \cdot n \, da$

$H = \frac{1}{\mu_0} B$

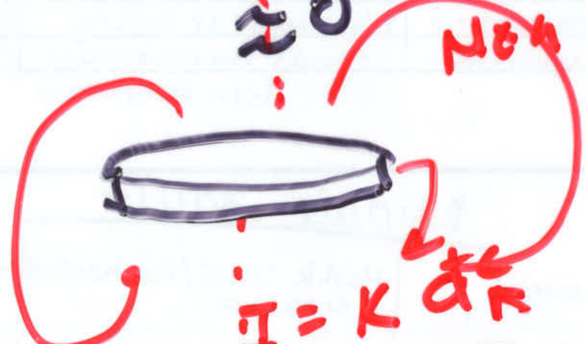


$B \overset{\leq}{=} \mu_0 K$

$= \mu_0 M$

$H = \underbrace{\frac{1}{\mu_0} B - M}_{\approx 0}$

$M a$



$H \approx 0$

$I = K d_R$

$\frac{\mu_0 I}{2R} \approx 0$

$\frac{\mu_0 K d}{2R} \approx B$

$B \approx 0$

$H = -M$

$E = \frac{\sigma}{\varepsilon_0}$

$\cdot\uparrow M\cdot a$

$$\int -\nabla\cdot M\, dV + \oint M\cdot n\, da$$

$$-\nabla\cdot M - \oint M\cdot n\, da = 0$$

$$\nabla\cdot D = \rho_f$$

$$\nabla\times H = J_f$$

$$\oint D\cdot n\, da = Q_f$$

$$\oint E\cdot n\, da = \frac{Q_{f+b}}{\varepsilon_0}$$

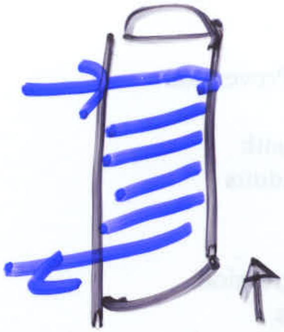
$$\int \nabla\times H\, da = \int J_f\, da = I_{enc}$$

$$\oint \vec{H}\cdot d\vec{l} = \underbrace{I_{enc}}_{\text{free}}$$

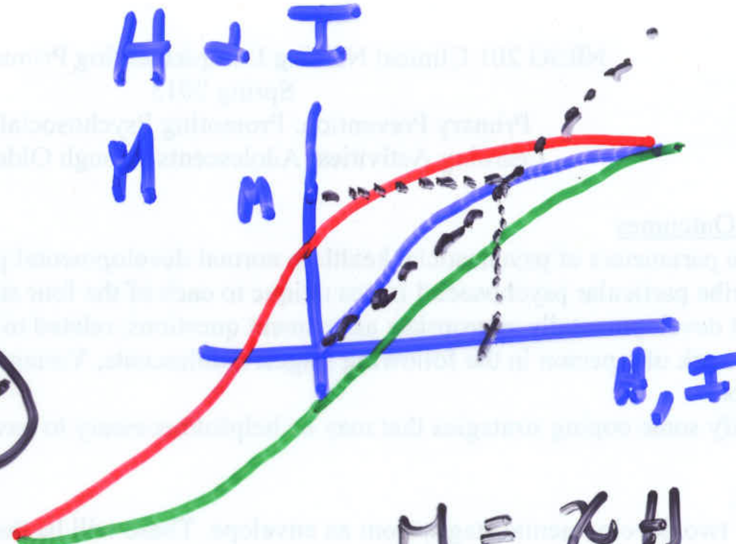
$\chi = 0$ ← non linear

$\chi \sim 1{,}000{,}000$

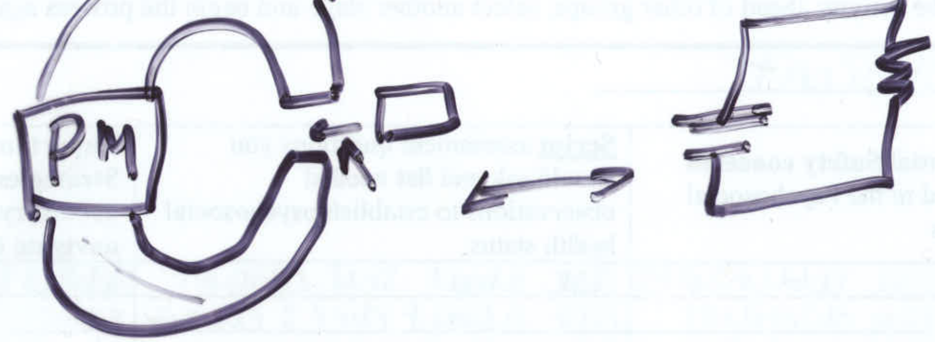
↳ M H



$H \propto I$



$M = \chi H$



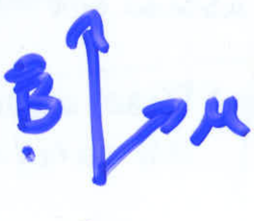
diamagnetism — $\vec{m} = -k\vec{B}$

→ QM Mechan $= 0$

$F = m\omega^2 R$

reduce

increase



$PE = -\mu \cdot B$

Ferromagnetism — domains