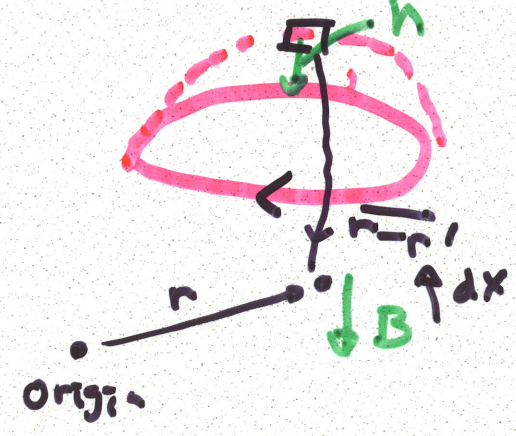
Solid Angle: 4π steradian = 100%

for sphere: $\Omega = \dfrac{\text{Area}}{R^2} = 2\pi(1-\cos\theta)$ (for area surrounding north pole)



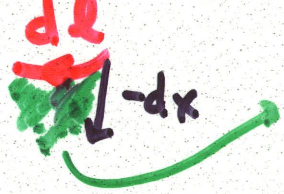
$$\Omega = \int \frac{\hat{n}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dA$$

area adjusted by $\cos\theta$ from dot product

Consider change in solid angle when observer moves distance $d\vec{x}$ $\leftrightarrow$ circuit moves $-d\vec{x}$

ribbon of addition area (close with previous area)

new location wire



$-\vec{dx}\times\vec{dl}$

$$-dx\cdot \overbrace{d\vec{l}\times(\vec{r}-\vec{r}')}$$

$$d\Omega = \int \frac{(-\vec{dx}\times\vec{dl})\cdot(\vec{r}-\vec{r}')}{|r-r'|^3}$$

$\nabla\Omega\cdot dx$ "

$$= -dx\cdot \underbrace{\int \frac{dl\times(r-r')}{|r-r'|^3}}_{B\frac{4\pi}{\mu_0 I}}$$

so:

$$-\frac{\mu_0 I}{4\pi}\vec{\nabla}\Omega = \vec{B}$$

multipole expansion

$$\vec{A} = \frac{\mu_0 I}{4\pi}\oint \frac{d\vec{r}'}{|\vec{r}-\vec{r}'|} \underset{r\to\infty}{\approx} \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

magnetic dipole

$$\vec{m} \equiv I\,\underbrace{\frac{1}{2}\oint \vec{r}'\times d\vec{r}'}_{\text{Area}}$$

Note: I've defined Ω with $\vec{r}-\vec{r}'$ vectors which is opposite of text book. I wanted ~~[illegible]~~ Ω to [illegible]

Forces & Torques

$q\vec{E} = \vec{F}$

$q\vec{v} \times \vec{B} = \vec{F}$

$\vec{m} \leftarrow \vec{p}$

$\vec{\tau} = \vec{p \times E}$

$\vec{F} = 0$

$PE = -\vec{p} \cdot \vec{E}$

$F = -\nabla PE$

1. Trust?
2. Love Tech*
3. Learn why

$B \leftarrow \vec{E}$

Prove

$\vec{A} \cdot \vec{B} = A_x B_x + \dots + A_z B_z = \sum A_i B_i$

$\vec{A} \times \vec{B} = (A_y^2 B_z^3 - A_z B_y, \dots$ $\overset{1\,3\,2}{\sum} \epsilon_{ijk} A_j B_k$

$R\vec{A} \times R\vec{B} = R(A \times B)$

vector

Mechan

$\epsilon_{ijk} = \begin{cases} 0 \\ +1 & 123,\ 312,\ 231 \\ -1 \end{cases}$

$(-)^{\sigma}$

$\epsilon_{213} = -1$

$\det(A) = \sum_{\sigma \in S_N} (-)^{\sigma} A_{1\sigma 1} A_{2\sigma 2} A_{3\sigma 3}$

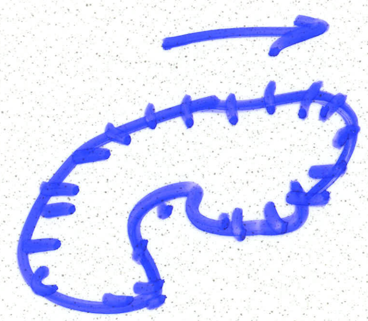
$(A \times B)_a = \sum_{jk} \epsilon_{ajk} A_j B_k$

$(\vec{A \times B}) = \sum_{a j k} \hat{e}_a \epsilon_{ajk} A_j B_k$

$$A\times(B\times C) = \vec{B}(A\cdot C) - \vec{C}(A\cdot B)$$

$\epsilon \qquad \epsilon_i \qquad \delta$

$$d\vec{F} = I\, d\vec{\ell}\times\vec{B}$$



$$\vec{F} = \oint I\, d\vec{\ell}\times\vec{B}$$

$$= I\underbrace{\left(\oint d\vec{\ell}\right)}_{=0}\times\vec{B}$$

$$\vec{\tau} = I\oint \vec{r}\times(d\vec{r}\times\vec{B})$$

$$= I\oint d\vec{r}(\underbrace{\vec{r}\cdot\vec{B}}_{\nabla\phi}) - \vec{B}(\overbrace{\vec{r}}^{\nabla\frac{1}{2}r^2}\cdot d\vec{r})^{0}$$

1.2.1 p22

$$\oint \phi\, d\vec{\ell} = \int A\times\underline{\underline{\nabla\phi}}\, da$$

$$\int \nabla\phi\cdot dr = \phi_f - \phi_i$$

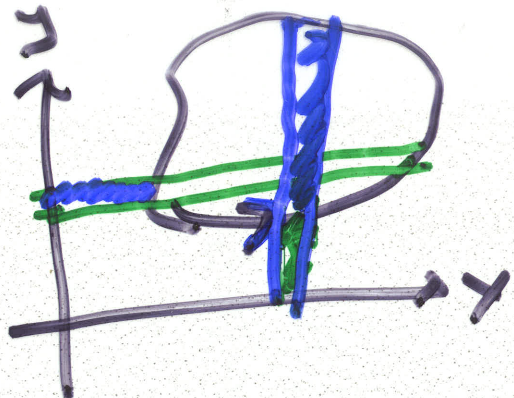
$$d\left(\tfrac{1}{2}r^2\right)$$

$$\frac{\int d\vec{A}\times B}{\vec{m}}$$

$xB_x + yB_y \to \vec{B}$

$$\tau_i = I\underbrace{\oint dr_i\, r_j}\, B_j$$

$$\oint dx\, y \qquad \oint dy\, x$$



$$\int dy\, x = A$$

$$\int dx\, y = -A$$

$$0 = \oint d(r_i r_j) = \oint dr_i\, r_j + \int r_i\, dr_j$$

$$\tau_i = I \oint \underbrace{dr_i\, r_j}_{\frac{1}{2} dr_i r_j - \frac{1}{2} r_i dr_j} B_j$$

$$= \frac{1}{2} I \int \left(dr_i\, r_j - r_i\, dr_j \right) B_j$$

$$\vec{\tau} = \frac{1}{2} I \int \vec{dr}\,(r \cdot B) - \vec{r}\,(dr \cdot B)$$

eye

$$\vec{B} \quad C \cdot A \quad C\, B \cdot A$$

$$= \frac{1}{2} I \int B \times (dr \times r)$$

$$= \underbrace{\frac{1}{2} I \int (r \times dr)}_{\vec{m}} \times \vec{B}$$

$$\vec{A} = \frac{1}{2}\int \overrightarrow{r \times dr}$$

$$A_x = \frac{1}{2}\int y\,dz - z\,dy$$

$$\longrightarrow \quad F = -\nabla PE \qquad PE = -\vec{m}\cdot\vec{B}$$

$$= \nabla\, \overrightarrow{m\cdot B} = m\cdot\nabla B$$

$$\nabla\cdot B$$

$$\partial_x\,(m_x B_x + \cdots + m_z B_z)$$

$$m_x\,\partial_x B_x + \cdots + m_z\,\partial_x B_z$$

$$\vec{F} = I \oint d\vec{r} \times \left(\vec{B}_0 + (\vec{r}\cdot\nabla)\vec{B} \cdots\right)$$

$$f(\vec{r}_0 + \vec{\Delta r}) = f(\vec{r}_0) + \vec{\Delta r}\cdot\vec{\nabla} f$$

$$= I \oint \vec{dr} \times \left[(r\cdot\nabla)\vec{B}\right]$$

$$F_i = I \oint \epsilon_{ijk}\, \underbrace{dr_j\, r_a}_{m_b\,\epsilon_{baj}}\, \partial_a B_k$$

$$\partial_a$$

$$= \epsilon_{ijk}\, \boxed{\epsilon_{jba}\, m_b\, \partial_a} B_k$$

$$(A \times B)_i = \epsilon_{ijk} A_j B_k$$

$$\square = m \times \nabla$$

$$F = (m \times \nabla) \times B$$

$$= B \times (\nabla \times m)$$

$$= \nabla(m \cdot B) - m \overbrace{\nabla \cdot B}^{0}$$

$$PE = -m \cdot B$$

$$\boxed{\sum_{i=1}^{3} \epsilon_{ijk} \epsilon_{iab} = \delta_{ja}\delta_{kb} - \delta_{jb}\delta_{ka}}$$