Bound Charge: $\rho_B = -\vec{\nabla}\cdot\vec{P}$ $\quad \sigma_B = \vec{P}\cdot\hat{n}$

$\vec{P}$: polarization: net dipole per volume

linear dielectric: $\vec{P} = \varepsilon_0 \chi \vec{E}$

$\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \underbrace{(1+\chi)}_{\text{dielectric constant } K} \vec{E}$

$\vec{\nabla}\cdot\vec{D} = \rho_f$ $\qquad \vec{\nabla}\cdot\vec{E} = \dfrac{\rho_f + \rho_B}{\varepsilon_0}$

$\vec{\nabla}\times\vec{D} \neq 0$ at boundary $\qquad \vec{\nabla}\times\vec{E} = 0$

$\oint \vec{D}\cdot\hat{n}\, dA = Q_f$ $\qquad \oint \vec{E}\cdot\hat{n}\, dA = Q_f + Q_B$

BC $\Delta D_n = \sigma_f$ $\qquad \Delta E_t = 0$

Note: linear dielectric $\Rightarrow \rho_B \propto \rho_f \Rightarrow$ $\rho_f = 0$ still allows Laplace

Unusual Case of material with $\vec{P}$ = const

"penny"

$\downarrow E \approx \dfrac{P}{\varepsilon_0}$

D small

outside E & D small

pencil

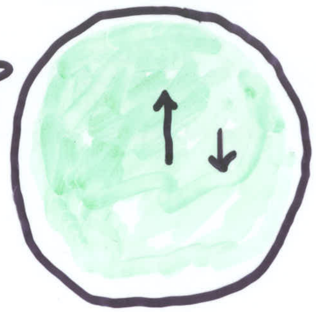
$Q = PA$

$D \approx P$

$-Q$

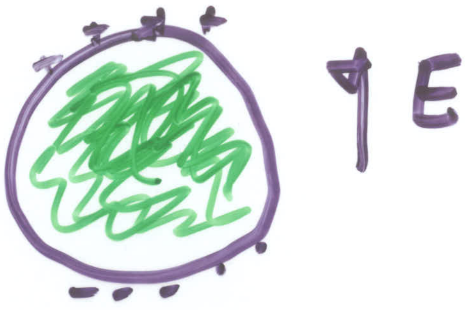
$\sigma_B = P\cos\theta$

$D = \frac{2}{3}P$



$E = \dfrac{-P}{3\varepsilon_0}$

$p = \frac{4}{3}\pi R^3 P$



$$\phi = \sum\Big(\underset{\uparrow}{A_n} r^{n} + \underset{\substack{\uparrow\\ \text{not}\\ \text{inside}}}{\frac{C_n}{r^{n+1}}}\Big) P_n$$

$$\phi = -E_0 z$$
$$= -E_0 r P_1 \qquad + \sum \frac{C_n}{r^{n+1}} P_n$$

$$\phi_{in} = \sum A_n r^n P_n$$

$$\phi_{out} = \left(-E_0 r + \frac{C_1}{r^2}\right) P_1$$

$$\hookleftarrow \Delta E_t = 0$$

$$\phi_{in} = \phi_{out} \Big|_{r=R}$$

$$D_n = -\partial_r \phi \, \varepsilon_0 K$$

$$\Delta D_n = 0$$

$$\vec{A} = \sum A_i \hat{e}_i \rightarrow \vec{A} \cdot \hat{e}_k = A_k \hat{e}_k \cdot \hat{e}_k$$

$$\phi_{in}\Big|_{r=R} = \sum A_n R^n P_n \qquad \sum \frac{C_n}{R^{n+1}} P_n$$

$$\phi_{out}\Big|_{r=R} = \left(-E_0 R + \frac{C_1}{R^2}\right) P_1 +$$

$\underline{n=1}$ $$A_1 R^1 = \left(-E_1 R + \frac{C_1}{R^2}\right)$$

$\underline{n \neq 1}$ $$A_n R^n = \frac{C_n}{R^{n+1}}$$

$\to 0$

$\underline{\underline{r=R}}$

$$\varepsilon_0 K \underset{\underset{-\partial_r \phi}{\uparrow}}{E_r^{in}} = \varepsilon_0 \cancel{K} \underset{\hat{}\ K=1 \text{ outside}}{E_r^{out}}$$

$$K \sum n A_n R^{n-1} P_n$$

$$= \left(-E_0 - \frac{2C_1}{R^3}\right) P_1 +$$

$$\sum -(n+1) \frac{C_n}{R^{n+2}} P_n$$

$\underline{n=1}$ $$K A_1 = \left(-E_0 - \frac{2C_1}{R^3}\right)$$

$\underline{n \neq 1}$ $$K n A_n R^{n-1} = -(n+1) \frac{C_n}{R^{n+2}}$$

$$ax + by = 0$$
$$cx + dy = 0$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$0 = C_n - A_n R^{2n+1}$$

$$0 = (n+1)\underset{\overset{\uparrow}{x}}{C_n} + K n \underset{\overset{\uparrow}{y}}{\underline{A_n R^{2n+1}}}$$

$$0 = 1x - y$$

$$0 = (n+1)x + Kn y$$

$$\det = Kn + (n+1) > 0$$

$\overline{n=1}$

$$A_1 - \frac{C_1}{R^3} = -E_0 \quad \times 2$$

$$KA_1 + \frac{2C_1}{R^3} = -E_0$$

$\longleftarrow 0$

$$2A_1 - \frac{2C_1}{R^3} = -2E_0$$

$$KA_1 + \frac{2C_1}{R^3} = -E_0$$

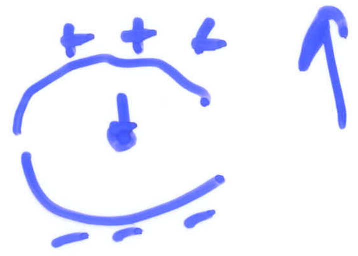
---

$$(K+2)A_1 = -3E_0$$

$$A_1 = \frac{-3}{(K+1)} E_0$$

$$\Phi_{in} = A_1 r \underset{\underset{\cos}{\uparrow}}{P_1} = -\frac{3E_0}{K+2} z$$

$$\vec{E} = \frac{3E_0}{K+2}\hat{k} \quad \hat{z}$$



$K \to 1$ nothing there

$K \to \infty$ "conductor"

$$\frac{-3}{(K+2)} E_0 \swarrow$$

$$\frac{2C_1}{R^3} = -E_0 - KA_1$$

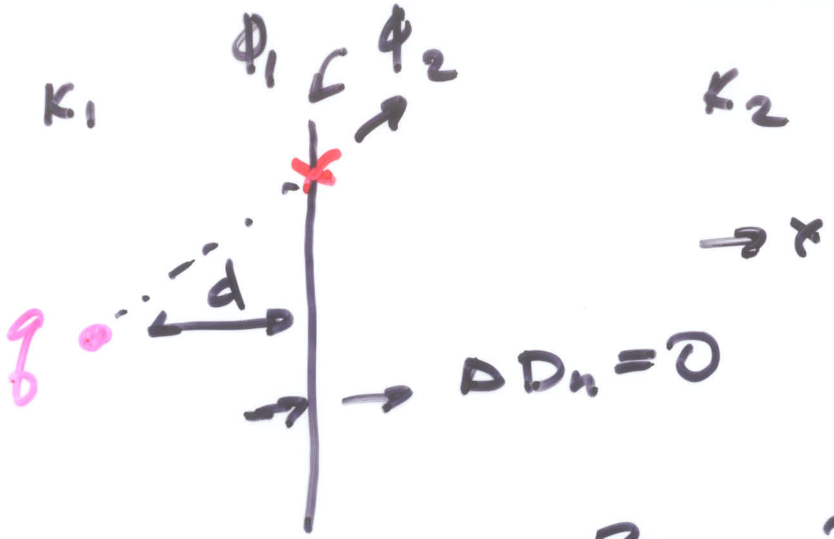
$$= \left(\frac{3K}{K+2} - 1\right) E_0$$

$$= \frac{2K-2}{K+2} E_0$$

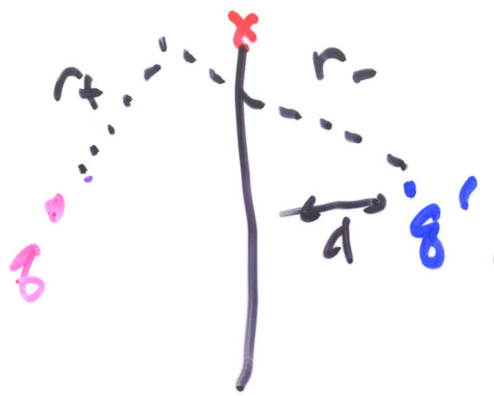
$$= \frac{2(K-1)}{K+2} E_0$$

$$\to C_1 = \frac{(K-1)}{(K+2)} E_0 R^3$$

# Image

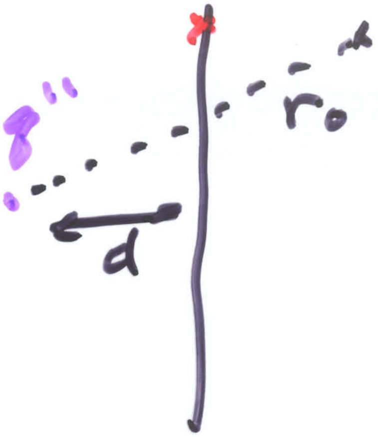


In region 1



$$\phi = \frac{1}{4\pi\varepsilon_1}\left(\frac{q}{r_+} + \frac{q'}{r_-}\right)$$

In Region 2



$$\phi = \frac{1}{4\pi\varepsilon_2}\,\frac{q''}{r_0}$$

on boundary $r_+ = r_- = r_0$

$$\phi_1 = \phi_2 \rightarrow \frac{q+q'}{\varepsilon_1} = \frac{q''}{\varepsilon_2}$$

$\Delta D_n = 0$

$$\varepsilon_1(-\partial_x \phi_1) = \varepsilon_2(-\partial_x \phi_2)$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{|r-r'|} \rightarrow E = \frac{1}{4\pi\varepsilon_0}\,\frac{q(\vec{r}-\vec{r}')}{|r-r'|^3}$$

$E_x$

$$\underbrace{\varepsilon_1 \times \frac{1}{4\pi\varepsilon_1}\left[\frac{q\,d}{r^3} - \frac{q' d}{r^3}\right]}_{E_{x1}\ @\ \text{Bound}} = \varepsilon_2 E_{x2}$$

$$\frac{1}{4\pi\varepsilon_2}\,\frac{q'' d}{r^3}$$

$$q - q' = q''$$

$$\frac{q + q'}{\varepsilon_1} = \frac{q''}{\varepsilon_2} \times \varepsilon_1$$

$$q - q' = q''$$

$$q + q' = \frac{\varepsilon_1}{\varepsilon_2} q''$$

$$2q = \left(1 + \frac{\varepsilon_1}{\varepsilon_2}\right) q''$$

$$\varepsilon_i = \varepsilon_0 K_i$$

$$\frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} q = q''$$

$$\shortparallel$$

$$\frac{2K_2}{K_1 + K_2} q$$

$$q' = q - q''$$

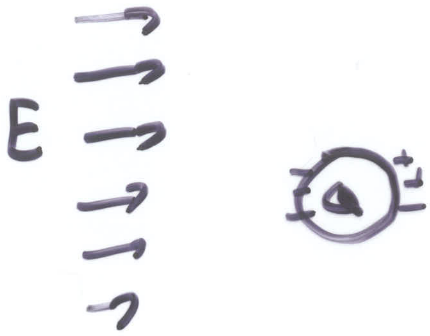
$$= \left(1 - \frac{2K_2}{K_1 + K_2}\right) q$$

$$= \left(\frac{K_1 - K_2}{K_1 + K_2}\right) q$$

$K_1 = K_2$

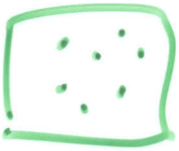
$K_2 \to \infty$

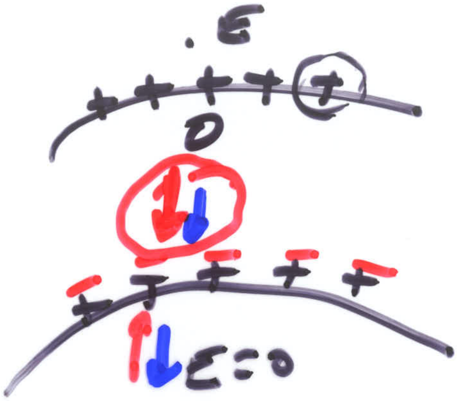
# Force conductor inside dielectric



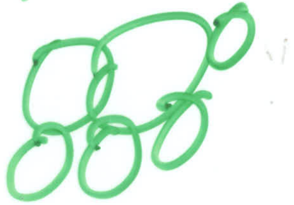
$$\vec{F} = \frac{1}{2}\int \vec{E}\,\sigma\, dA$$

$\frac{1}{2} E = \underline{\text{avg } E}$ ✓





$\chi \sim \frac{\text{volume atm}}{\text{total}}$



$\vec{E}$ inside

$H_2O$ $\underline{\text{avg}}$