Fourier's Trick : $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$A(x)$ $\qquad$ $e_i(x)$

$\vec{A} \cdot \hat{k} = A_z \hat{k} \cdot \hat{k}$

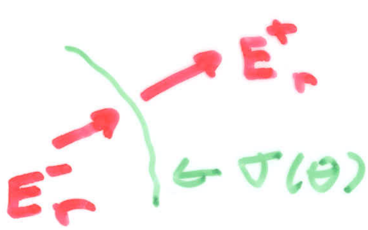
$$\int A(x)\, e_i(x)\, dx = A_i \int e_i^2\, dx$$

$$\phi = \sum A_n \phi_n \leftarrow \sin\left(\frac{\pi n}{b} y\right) \frac{\sinh\left(\frac{\pi n}{b} x\right)}{\sinh\left(\frac{\pi n}{b} a\right)}$$

$$A_n = \int_0^b V(y) \sin\left(\frac{\pi n}{b} y\right) \Big/ \tfrac{1}{2} b$$

$$\phi = \sum \left(A_n r^n + \frac{C_n}{r^{n+1}} \right) P_n(\cos\theta)$$

inside usually $\qquad$ outside usually



$$\sigma = \varepsilon_0 (E_r^+ - E_r^-)$$
$$= \varepsilon_0 (\partial_r \phi^- - \partial_r \phi^+)$$

$r < R$ $\quad \phi^- = \sum A_n r^n P_n$

$r > R$ $\quad \phi^+ = \sum \frac{C_n}{r^{n+1}} P_n$

agree at $r = R$

$$A_n R^n = \frac{C_n}{R^{n+1}}$$

$$\sigma(\theta) = \sum \varepsilon_0 \left(n A_n R^{n-1} + \frac{(n+1) C_n}{R^{n+2}} \right) P_n$$

$$\hookrightarrow (2n+1) A_n R^{n-1}$$

$$\int_{-1}^{1} \sigma(\theta)\, P_m(c)\, dc = \varepsilon_0 (2m+1) A_m R^{m-1}$$

$$\int P_m P_m \, dc = \underbrace{}_{\frac{2}{2m+1}}$$

$$= 2 \varepsilon_0 A_m R^{m-1}$$

Ex 3

$$\phi(z) = \sum a_n z^n$$

$$\underset{\text{inside}}{\phi(r, c)} = \sum A_n \underbrace{r^n P_n(c)}_{z^n}$$

$$\theta = 0 \rightarrow c = 1 \rightarrow P_n(1) = 1$$

$$r = z \qquad P_2 = \frac{3}{2} c^2 - \frac{1}{2}$$

$$A_n = a_n \qquad P_1 = c$$

$$P_n(-1) = (-1)^n$$

R

Q

$$\phi(z) = \frac{Q}{4\pi\varepsilon_0 \sqrt{z^2 + R^2}}$$

$$= \frac{Q}{4\pi\varepsilon_0 R \sqrt{1 + z^2/R^2}}$$

$$(1-x)^{-\alpha} = \sum \frac{(\alpha)_n}{n!} x^n$$

$$\left(1 + \frac{z^2}{R^2}\right)^{-1/2} = \sum \frac{(\frac{1}{2})_n}{n!} \left(\frac{-z^2}{R^2}\right)^n$$

$$\phi(z) = \frac{Q}{4\pi\epsilon_0 R} \sum \frac{(\frac{1}{2})_n}{n!} \left(\frac{-z^2}{R^2}\right)^n$$

$$= \sum A_n z^n \qquad A_{odd} = 0$$

$$\frac{Q}{4\pi\epsilon_0 R} \frac{(\frac{1}{2})_n}{n!} \frac{(-1)^n}{R^{2n}} = A_{2n}$$

$$\left(\tfrac{1}{2}\right)_3 = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2}$$

$$\left(\tfrac{1}{2}\right)_n = \underbrace{\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \cdots}_{n \text{ terms}}$$

$$(1)_n = 1 \cdots n = n!$$

P_1

$rc = rP_1$

$$\phi = -E_0 z$$

$$\nabla\phi = E_0 \hat{k}$$

Cu

V const

R

$$\phi = \sum \frac{c_n}{r^{n+1}} P_n - E_0 r P_1$$

$$\underset{\text{const}}{=} \phi(R, c) = \sum \frac{c_n}{R^{n+1}} P_n - E_0 R P_1$$

$n > 1$ $\quad \int \underset{P_0}{\text{const}} \cdot P_m \, dc = \dfrac{C_m}{R^{m+1}} \dfrac{2}{2m+1}$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad \| \; 0$

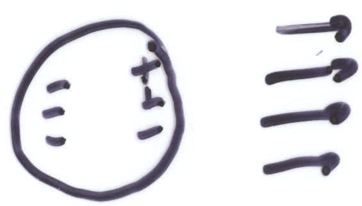
$$\phi = \left(\frac{C_1}{r^2} - E_0 r \right) P_1 \quad + \cancel{\text{const}}$$

ground Sphere $\to \phi = 0$

$$\frac{C_1}{R^2} = E_0 R$$

$$C_1 = E_0 R^3$$

3-12



$$\phi = -E_0 x$$
$$= -E_0 r \cos\phi$$

θ in book

cylinder: $\nabla^2 \phi = 0$

volt

$$\frac{1}{r}\partial_r(r\,\partial_r \phi) + \frac{1}{r^2}\partial_\Theta^2 \phi + \partial_z^2 \phi = 0$$

does not depend on z

$$\phi = \Theta(\theta)\, R(r)$$

$$\Theta\, \frac{1}{r}(r R')' + \frac{R}{r^2}\Theta'' = 0$$

$$\frac{r(r R')'}{R} = \frac{-\Theta''}{\Theta} = m^2$$

$$\Theta = \begin{cases} \sin(m\theta) \\ \cos(m\theta) \end{cases}$$

$$R = r^q$$

$$R' = q\, r^{q-1}$$

$$r R' = q\, r^q$$

$$(\ )' = q^2 r^{q-1}$$

$$r(\ )' = q^2 r^q$$

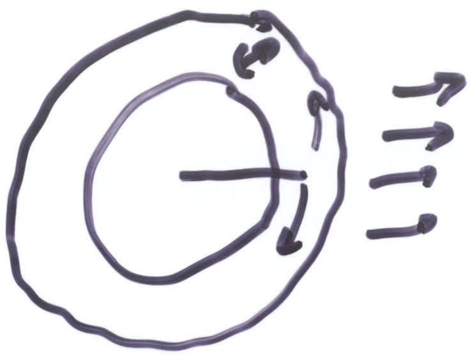
$$r(r R')' = m^2 R$$

$$q^2 r^q = m^2 r^q$$

$$q = \pm m$$

$$\phi = \sum \left(A_m r^m + \frac{C_m}{r^m} \right) \left(a_m \cos(m\theta) + b_m \sin(m\theta) \right)$$

$$\int_0^{2\pi} \cos(m\theta)\cos(n\theta) = \delta_{mn}\, \pi$$



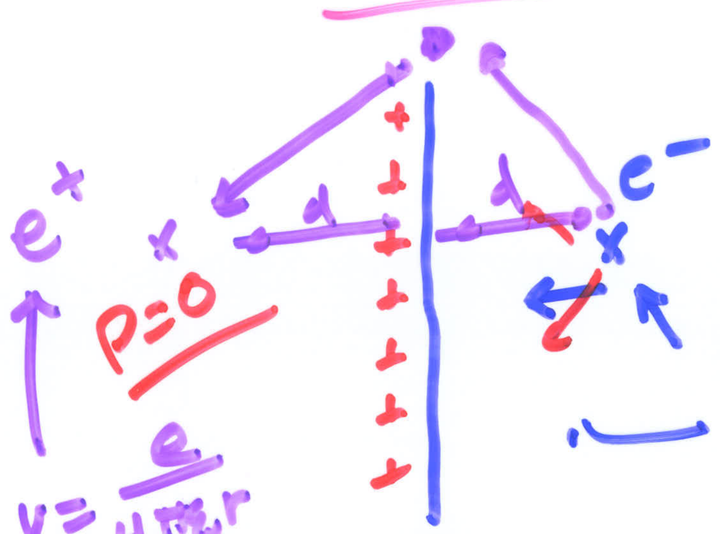
cos

Fin

① set $\nabla^2 \phi = 0$

② set B.C.

Images



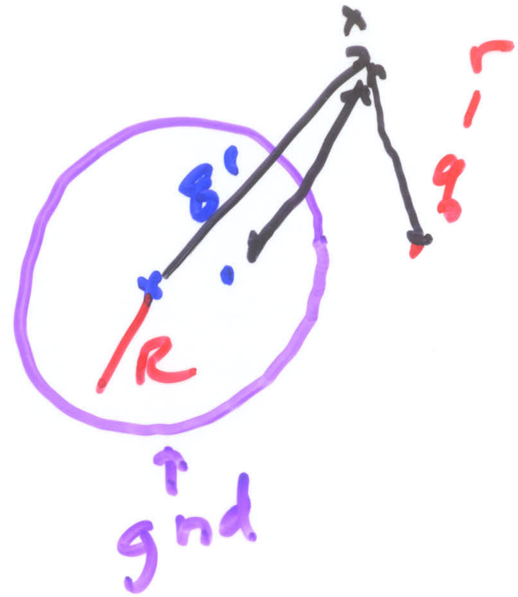
$\phi(\infty) = 0$

$$\phi = \frac{(-e)}{4\pi\varepsilon_0 r_-} + \frac{e}{4\pi\varepsilon_0 r_+}$$

$\phi (= 0$ ✓

$\phi = 0$

$$F = \frac{e^2}{4\pi\varepsilon_0 (2d)^2}$$

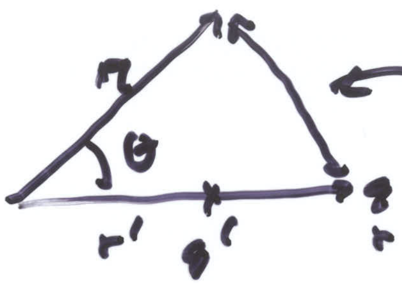


- $\nabla^2\phi = 0$
- ϕ BC

$\phi(r=R)=0$

$\phi(\infty)=0$

q' @ $r=r'$



$\sqrt{\pi^2+r^2-2\pi r\cos\theta}$

r'

$$\phi(\pi,\theta) = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q}{\sqrt{\pi^2+r^2-2\pi r\cos\theta}} + \frac{q'}{\sqrt{\pi^2+r'^2-2\pi r'\cos\theta}}\right\}$$

$$\phi(R,\theta)=0$$

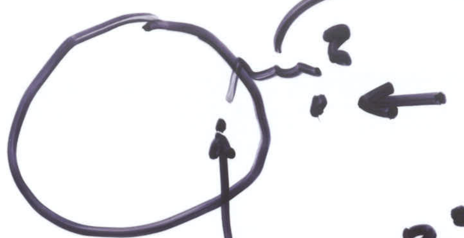
$$\frac{q}{\sqrt{R^2+r^2-2Rrc}} = \frac{-q'}{\sqrt{R^2+r'^2-2Rr'c}}$$

$\forall c$

if $\left.\begin{matrix} r = R+\delta \\ r' = R-\delta \quad q' = -q \end{matrix}\right]$ ✗

$$\frac{-q'}{R\sqrt{1+\frac{r'^2}{R^2}-2\frac{r'}{R}c}} = \frac{q}{r\sqrt{\frac{R^2}{r^2}+1-2\frac{R}{r}c}}$$

$$\left.\begin{aligned} \frac{r'}{R} &= \frac{R}{r} \\ -q' &= \frac{R}{r}\,q \end{aligned}\right\}\ \text{works!}$$



$$r - \frac{R^2}{r}$$

$$q' = \frac{R^2}{q\,r}$$

$$F = \frac{k\,q\,q'}{\left(r-\frac{R^2}{r}\right)^2} = \frac{-k\,q^2\,\frac{R}{r}}{\left(r-\frac{R^2}{r}\right)^2}$$

uncharged = $Q_{net} = 0$

vs

grounded = $V = 0$