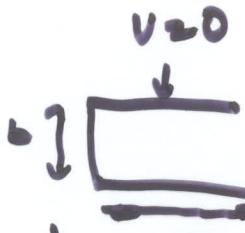


Fourier's Trick : $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$$A(x) \quad \vec{A} \cdot \vec{E} = A_z E_z \quad e_i(x)$$


$$\int A \sin e_i(x) dx = A_i \int e_i^2 dx$$

$$\phi = \sum A_n \phi_n \quad \sin\left(\frac{\pi n}{b} y\right) \quad \frac{\sinh\left(\frac{\pi n}{b} x\right)}{\sinh\left(\frac{\pi n}{b} a\right)}$$

$$A_n = \int_0^b V(y) \sin\left(\frac{\pi n}{b} y\right) / \frac{1}{2} b$$

$$\phi = \sum \left(A_n r^n + \frac{C_n}{r^{n+1}} \right) P_n (\cos \theta)$$

inside usually

outside usually

$$E_r^+ \quad \sigma = \epsilon_0 (E_r^+ - E_r^-)$$

$$E_r^- \quad \leftarrow \tau(\theta) \quad = \epsilon_0 (\lambda_r^+ - \lambda_r^-)$$

$$r < R \quad \phi = \sum A_n r^n P_n \quad \left. \begin{array}{l} \text{agree at } r=R \\ A_n R^n = \frac{C_n}{R^{n+1}} \end{array} \right\}$$

$$r > R \quad \phi^+ = \sum \frac{C_n}{r^{n+1}} P_n \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\tau(\theta) = \sum \left(n A_n R^{n-1} + \frac{(n+1) C_n}{R^{n+2}} \right) P_n$$

$$\downarrow (2n+1) A_n R^{n-1}$$

$$\int_{-1}^1 \sigma(\theta) P_m(c) dc = \varepsilon_0 (2m+1) A_m R^{m-1}$$

\hookrightarrow $\underbrace{\int P_m P_m dc}_{\frac{2}{2m+1}}$
 $= 2 \varepsilon_0 A_m R^{m-1}$

θ

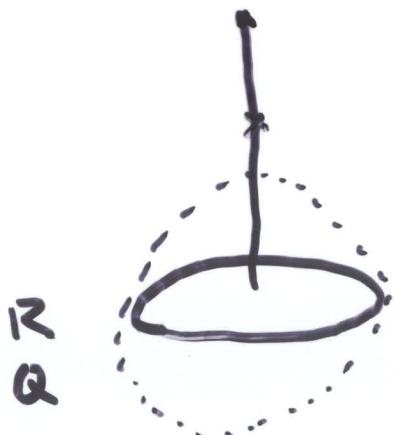
$E \times 3$

$$\phi(z) = \sum q_n z^n$$

$\hookrightarrow \phi(r, c) = \sum A_n r^n \underbrace{P_n(c)}$

$$\theta = 0 \rightarrow c = 1 \rightarrow P_n(1) = 1$$

$r = z$ $P_2 = \frac{3}{2} c^2 - \frac{1}{2}$
 $A_n = q_n$ $P_1 = c$
 $P_1(-1) = (-1)^n$



$$\phi(z) = \frac{Q}{4\pi\varepsilon_0} \int \frac{dz'}{z'^2 + R^2}$$

$$= \frac{Q}{4\pi\varepsilon_0 R} \sqrt{1 + \frac{z^2}{R^2}}$$

$$(1-x)^{-\frac{1}{2}} = \sum \frac{(1)_n}{n!} x^n$$

$$(1 + \frac{z^2}{R^2})^{-1/2} = \sum \frac{(\frac{1}{2})_n}{n!} \left(\frac{-z^2}{R^2}\right)^n$$

$$\Phi(z) = \frac{Q}{4\pi\epsilon_0 R} \sum \frac{\left(\frac{1}{z}\right)_n}{n!} \left(-\frac{e}{R}\right)^n$$

$$= \sum A_n z^n \quad A_{\text{odd}} = 0$$

$$\frac{Q}{4\pi\epsilon_0 R} \frac{\left(\frac{1}{z}\right)_n}{n!} \frac{(-1)^n}{R^{2n}} = A_{2n}$$

$$\left(\frac{1}{z}\right)_3 = \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2}$$

$$\left(\frac{1}{z}\right)_n = \underbrace{\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \cdots}_{n \text{ terms}}$$

$$(1)_n = 1 \cdots n = n!$$

$\rightarrow \uparrow \uparrow \uparrow \uparrow$

$$\phi = -E_0 z$$

$$\nabla \phi = E_0 \hat{k}$$

$$P_1 \downarrow$$

$$rC \downarrow$$

$$r = rP_1$$



$$\phi = \sum \frac{c_n}{r^{n+1}} P_n$$

$$-E_0 r P_1$$

$$\phi(R, c) = \sum \frac{c_n}{R^{n+1}} P_n$$

$$\text{const} = -E_0 R P_1$$

$n \geq 1$

$$\int_{P_0}^{\text{Const}} \frac{C_m}{r^{m+1}} dr = \frac{C_m}{(m+1)} \frac{r^2}{2} \Big|_{P_0}^{\text{Const}}$$

$$\phi = \left(\frac{C_1}{r^2} - E_0 r \right) P_1 + \cancel{C_0 k}$$

ground Shore $\rightarrow \phi = 0$

$$\frac{C_1}{R^2} = E_0 R$$

$$C_1 = E_0 R^3$$

$3-12$



$$\begin{aligned} \phi &= -E_0 x \\ &= -E_0 r \cos \theta \end{aligned}$$

θ in book

Cylinder: $\nabla^2 \phi = 0$

$$\frac{1}{r} \partial_r (r \partial_r \phi) + \frac{1}{r^2} \partial_\theta^2 \phi + \partial_z^2 \phi = 0$$

volt

does not depend on z



$$\phi = \Theta(\theta) R(r)$$

$$\Theta \frac{1}{r} (r R')' + \frac{R}{r^2} \Theta'' = 0$$

$$r \frac{(r R')'}{R} = - \frac{\Theta''}{\Theta} = m^2$$

$$\Theta = \begin{cases} \sin(m\theta) \\ \cos(m\theta) \end{cases}$$

$$r (r R')' = m^2 R$$

$$a^2 r^q = m^2 r^q$$

$$q = \pm m$$

$$R = r^q$$

$$R' = q r^{q-1}$$

$$r R' = q r^q$$

$$(r')' = q^2 r^{q-1}$$

$$r (r')' = q^2 r^q$$

$$d = \sum (a_m r^m + \frac{c_m}{r^m})$$

$$\int_0^{2\pi} \cos(m\theta) \cos(n\theta) = \delta_{mn} \pi$$



Cos

Fin

Images

- ① S_{ext} $\nabla^e q = 0$ -
- ② S_{ext} B.C. -

$$\phi(\infty) = 0$$

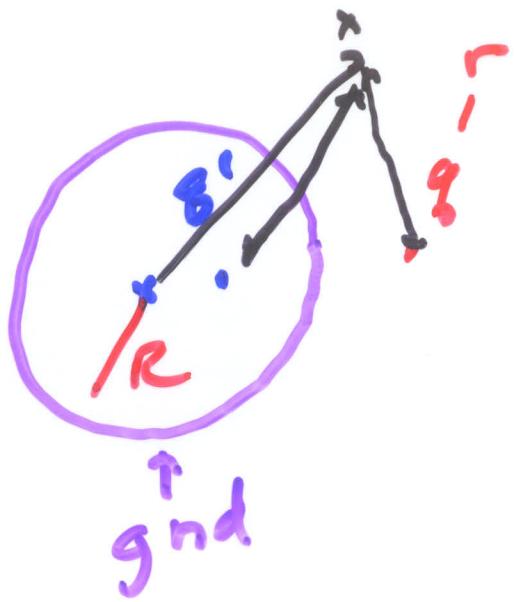
Diagram illustrating the electric field and potential distribution in a parallel plate capacitor. The left plate has charge density σ and voltage $V = \frac{e}{4\pi\epsilon_0 r}$. The right plate has charge density -2σ and voltage $V = -\frac{e}{4\pi\epsilon_0 r}$. The central vertical line represents the potential $\phi = 0$, with arrows indicating the direction of the electric field E .

Equation for potential ϕ :

$$\phi = \frac{(l-a)}{4\pi\epsilon_0 r_-} + \frac{a}{4\pi\epsilon_0 r_+}$$

Equation for force F :

$$F = \frac{e^2}{4\pi\epsilon_0 (2d)^2}$$

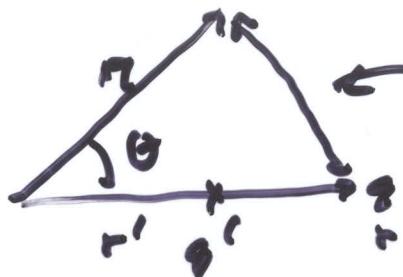


- $\nabla^2 \phi = 0$
- $\phi \text{ BC}$

$$\phi(r=R) = 0$$

$$\phi(\infty) = 0$$

$$\sigma' e^{-r/r'} \quad \overbrace{\qquad\qquad\qquad}$$



$$\sqrt{R^2 + r^2 - 2Rr \cos \theta} \quad \overbrace{\qquad\qquad\qquad} \\ r'$$

$$\phi(R, \theta) = \frac{1}{4\pi\epsilon_0} \left\{ \frac{\sigma}{\sqrt{R^2 + r^2 - 2Rr \cos \theta}} + \frac{\sigma'}{\sqrt{R^2 + r'^2 - 2Rr' \cos \theta}} \right\}$$

$$\frac{\sigma}{\sqrt{R^2 + r^2 - 2Rr \cos \theta}} = \frac{-\sigma'}{\sqrt{R^2 + r'^2 - 2Rr' \cos \theta}}$$

if $r = R + \delta$
 $r' = R - \delta$ $\sigma' = -\sigma$] \times

$$R \sqrt{1 + \frac{r^2}{R^2} - 2 \frac{r}{R} c} = r \sqrt{\frac{R^2}{r^2} + 1 - 2 \frac{R}{r} c}$$

$\frac{r'}{R} = \frac{r}{r}$

$-g' = \frac{r}{r} g$

works!

$$r - \frac{R^2}{r}$$

$$F = \frac{k q_0 q'}{(r - \frac{R^2}{r})^2}$$

uncharged $\Rightarrow Q_{net} = 0$ $= -\frac{k q^2 \frac{R}{r}}{(r - \frac{R^2}{r})^2}$

vs
grounded $\Rightarrow V = 0$

