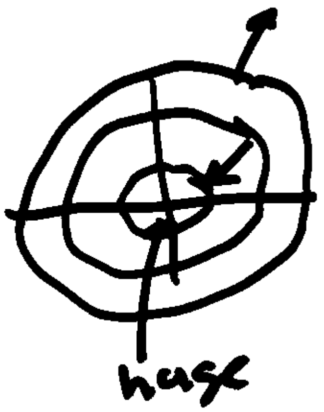
$\vec{r} = \langle x, y, z \rangle \qquad |\vec{r}| = r = \sqrt{x^2+y^2+z^2}$

$\vec{\nabla} \cdot \vec{r} = 3 \qquad \nabla \times \vec{r} = 0$

$\vec{\nabla} r = \langle \frac{1}{2}(\ )^{-1/2} 2x, \ , \ \rangle = \langle \frac{x}{r}$

$= \frac{\vec{r}}{r} = \hat{r}$



$\vec{\nabla} \frac{1}{r} =$

$\frac{1}{r} = [x^2+y^2+z^2]^{-1/2}$

$\partial_x \frac{1}{r} = \frac{-1}{2} [\ ]^{-3/2} 2x$

$= \frac{-x}{r^3}$

$\vec{\nabla} \frac{1}{r} = \frac{-\vec{r}}{r^3}$

$\frac{1}{|\vec{r}-\vec{r}_0|}$

$|\vec{r}-\vec{r}_0| = \sqrt{(x-x_0)^2+(y-y_0)^2 + (z-z_0)^2}$

$\partial_x \frac{1}{|r-r_0|} = -\frac{1}{2} (\ )^{-3/2} 2(x-x_0)$

$\vec{\nabla} \frac{1}{|r-r_0|} = \frac{-(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3}$

$$\nabla^2 = \nabla \cdot \vec{\nabla} = \partial_x^2 + \partial_y^2 + \partial_z^2$$

$$\nabla^2 \frac{1}{|r - r_0|} = \nabla \cdot \frac{-(\vec{r - r_0})}{|r - r_0|^3}$$

$$\frac{-(x - x_0)}{\left[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right]^{3/2}}$$

$$\partial_x \Rightarrow \quad \frac{-1}{[\ \ ]^{3/2}} - \frac{(x - x_0)\left(-\frac{3}{2}\right)}{[\ \ ]^{5/2}} 2(x - x_0)$$

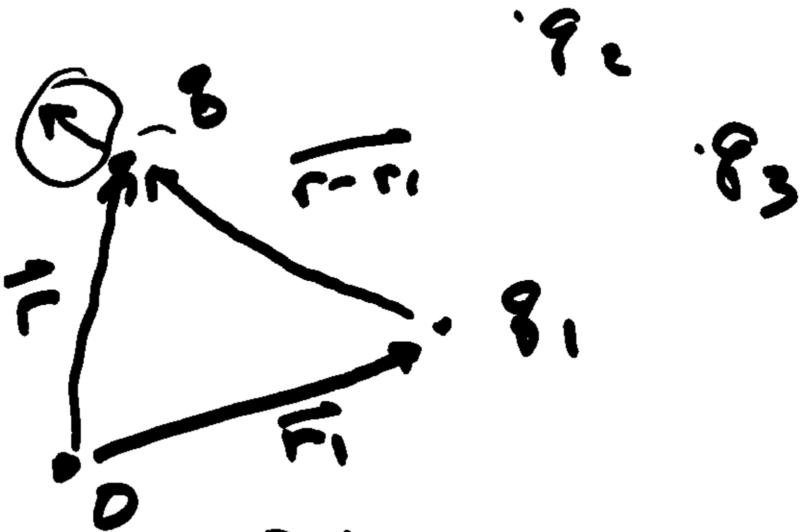
$$= \frac{-1}{[\ \ ]^{3/2}} + 3\frac{(x - x_0)^2}{[\ \ ]^{5/2}}$$

$$\downarrow$$

$$3\times \quad + 3\frac{\left((x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2\right)}{[\ \ ]^{5/2}}$$

$$= 0 \quad \text{if } [\ ] \neq 0$$

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$



$$\frac{\overline{r - r_1}}{|\overline{r - r_1}|^3} \frac{q q_1}{4\pi\epsilon_0}$$

$$F = mg$$

$$\vec{F}_q = \sum \frac{q q_i (\vec{r} - \vec{r}_i)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_i|^3}$$

$$= q \sum \frac{q_i (\vec{r} - \vec{r}_i)}{4\pi\epsilon_0 |\vec{r} - \vec{r}_i|^3} = \vec{E}(\vec{r})$$

test charge

source $\vec{r}'$

ρ ← charge density

$= \frac{Q}{V}$ volume

$\sigma = \frac{Q}{A}$ surface charge density

$\lambda = \frac{Q}{L}$ linear charge density

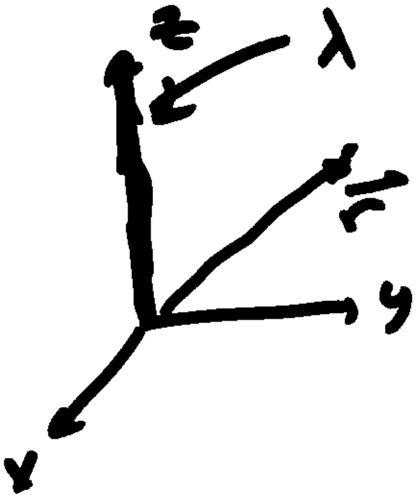
$q_i = \rho\, dV$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho\, dV\, (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

$\rho\, dV$

$\sigma\, dA$

$\lambda\, dx$



$$\vec r = \langle x, 0, z \rangle$$

$$\vec r\,' = \langle 0, 0, z' \rangle$$

$$\overline{r - r'} = \langle x, 0, z - z' \rangle$$

$$\vec E(\vec r) = \frac{1}{4\pi\varepsilon_0}\int_0^\infty dz' \, \frac{\lambda \langle x, 0, z - z' \rangle}{\sqrt{x^2 + (z - z')^2}^{\,3}}$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^\infty dz' \, \frac{x}{\sqrt{x^2 + (z - z')^2}^{\,3}}$$

$$E_z = \frac{\lambda}{4\pi\varepsilon_0}\int_0^\infty dz' \, \frac{z - z'}{\sqrt{x^2 + (z - z')^2}^{\,3}}$$

$z - z' = u$

$-dz' = du$

$$x \int_z^{-\infty} \frac{-1}{\sqrt{u^2 + x^2}^{\,3}}$$

$$= \frac{\cancel{x}}{x^2} \, \frac{-u}{\sqrt{u^2 + x^2}} \Bigg|_z^{-\infty}$$

$$= \frac{1}{x}\left(\frac{z}{\sqrt{z^2 + x^2}} - -1 \right)$$

Dwight 200.03 $\displaystyle\int \frac{dx}{\sqrt{x^2+a^2}^{\,3}} = \frac{1}{a^2}\,\frac{x}{\sqrt{x^2+a^2}}$

201.01 $\displaystyle\int \frac{x\,dx}{\sqrt{x^2+a^2}^{\,3}} = \frac{-1}{\sqrt{x^2+a^2}}$