1. An azimuthally symmetric voltage has been placed on the surface of the sphere:

$$V(\theta) = V_0 \cos^2\theta$$

where V_0 is a given constant. Find the resulting voltage ϕ inside and outside the sphere.

This is a rather long problem, so let me give you some initial help. Since the problem is azimuthally symmetric and ϕ must satisfy Laplace's equation, I know:

$$\phi_{\text{in}}(r,\theta) = \sum_{n=\text{even}} A_n r^n P_n(\cos\theta)$$

$$\phi_{\text{out}}(r,\theta) = \sum_{n=\text{even}} C_n r^{-n-1} P_n(\cos\theta)$$

where A_n and C_n are currently undetermined constants, ϕ_{in} gives ϕ for $r < R$, and ϕ_{out} gives ϕ for $r > R$.

(a) Explain why (words!) these particular formulas must hold. I.e., what is the general case and how/why does this problem simplify that general case.

(b) At the boundary (i.e., $r = R$) the following condition must hold:

$$\phi_{\text{in}}(R,\theta) = \phi_{\text{out}}(R,\theta) = V(\theta)$$

I conclude from this condition that, for every n,

$$A_n R^{2n+1} = C_n$$

Explain (words!) the basis for this conclusion. How do we go from one equation (which involves a sum of an infinite number of terms) to an infinite number of equations (one for every n)?

(c) The Mathematica command:

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Table[Integrate[c^2 LegendreP[n,c],{c,-1,1}],{n,0,10}]
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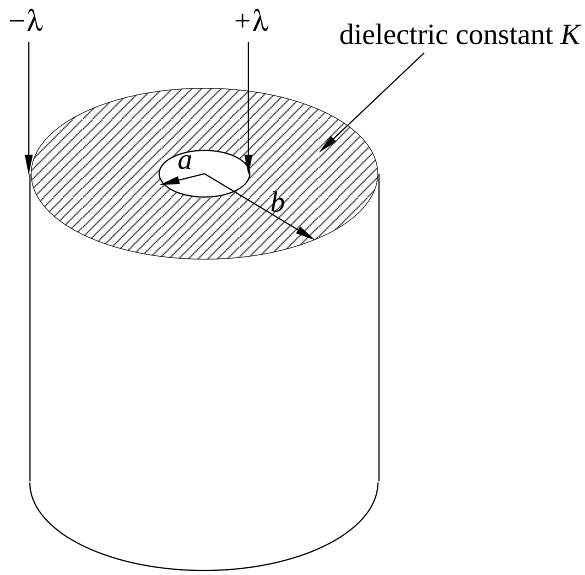
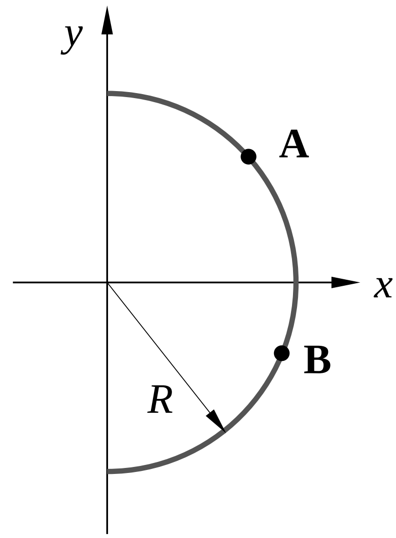
Produces the output: $\left\{\frac{2}{3}, 0, \frac{4}{15}, 0, 0, 0, 0, 0, 0, 0, 0\right\}$. Use this result to write down the first two non-zero terms (i.e., a couple of non-zero As and a couple of non-zero Cs) of the formulas for $\phi_{\text{in}}(r,\theta)$ and $\phi_{\text{out}}(r,\theta)$.

2. Consider two conducting coaxial infinite cylindrical shells. The volume between these shells is filled with dielectric (dielectric constant K); the remaining volume is vacuum. The inner cylinder (of radius a) carries a net charge per length of $+\lambda$ spread evenly around its surface. The outer shell (of radius b) carries the opposite net charge per length. (Note: the same charge per length with different surface areas means the σ are the not same.)

   (a) Use Gauss' Law to find the electric field in the three regions: (*i*) $r < a$, (*ii*) $a < r < b$, and (*iii*) $b < r$. Remember to report the direction of $\mathbf{E}$ and any symmetry arguments you have used.

   (b) Integrate $\mathbf{E}$ to find formulae for electric potential in those three regions (assume that the electric potential is zero at the center of the cylinder and make sure ϕ is continuous across boundaries).





3. A half circle (radius R) is made of a wire with uniform charge per length λ. Find the electric potential (voltage) and electric field vector at the center of the circle. Report what you are using for $\mathbf{r}$, $\mathbf{r}'$ and dq. Directly on the figure above right: draw the vector $\mathbf{r}'$ for the bit of charge labeled **A** and draw the vector $\mathbf{r}$-$\mathbf{r}'$ for the charge labeled **B**. Show all steps required to connect the general formulas for ϕ and $\mathbf{E}$ to the integrals you finally evaluate. (The integrals should not be hard to do.)

4. Consider the following three image charge problems:

   (a) A point charge q is a distance d from an infinite conducting, grounded plane

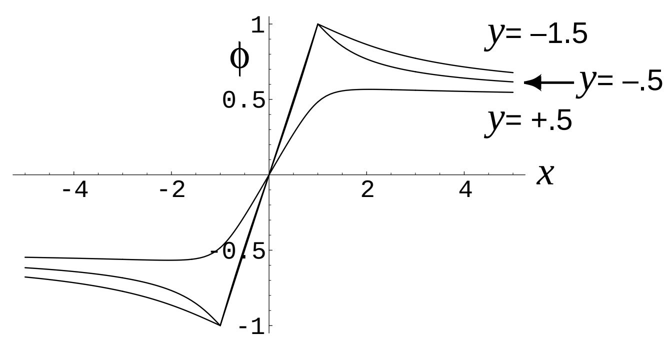
   (b) A point charge q is a distance d from the center of a conducting, uncharged sphere (radius R; $d > R$)

   (c) A uniform line charge λ is coaxial to and a distance d from the center of an infinite conducting cylinder (radius R; $d > R$) carrying a net charge per length of $-\lambda$.

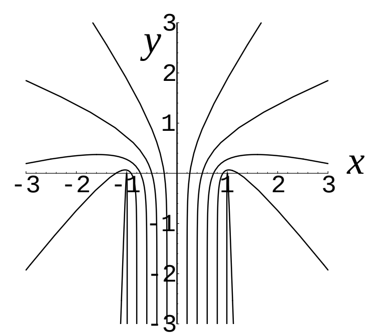
   For each of these problems draw an appropriate set of image charges. (No proofs required, simply apply the results we proved in class.) In each case, report a formula for the force on the charge. Carefully note in the above the words "grounded" and "uncharged". What is the difference?

5. The middle figure shows a contour graph of the potential (voltage) ϕ near the edge of a pair of parallel conducting plates forming a capacitor. The plates are infinite in the z directions (perpendicular to this page) and infinite in the y^- direction (down the page). The plate at $x = 1$ has potential $+1$; the plate at $x = -1$ has potential -1. The 11 contours shown are for $\phi = \{-.99, -.8, , -.6, \ldots, +.6, +.8, +.99\}$. The potential ϕ as a function of x is plotted at several different y values: $y = +.5$ (just above the capacitor edge); and $y = -.5$ and $y = -1.5$ (in part, inside the capacitor). Note that inside the capacitor the potential is nearly a linear function of x.

   (a) From the ϕ vs. x plots I conclude that the surface charge densities inside the capacitor at $y = -.5$ and $y = -1.5$ are much the same, whereas the surface charge density on the outside surface of the ($x = 1$) plate at $y = -.5$ is larger than surface charge density on the outside surface of the plate at $y = -1.5$. Explain! Hint: think $\mathbf{E}$!



   (b) Directly on the contour plot draw several $\vec{\mathbf{E}}$-field lines, including lines that go through the points: $(0, -2)$, $(0, 0)$, and $(0, 2)$. Be sure to include the direction of each $\vec{\mathbf{E}}$-field line. The electric field at $(0, 2)$ is smaller than that at $(0, -2)$. Explain how the contour plot displays this fact.



   (c) The bottom figure represents a cut through the capacitor plates in the vicinity of its top. Directly on this diagram, show the direction of the electric field at the points: $(-1.01, -1)$, $(-.99, -1)$, $(1.01, -1)$ and $(.99, -1)$ (i.e., slightly to the right and left of each plate at $y = -1$). Using little + and – signs show how the electric charge is distributed in/on the plates.