The S' frame moves with a velocity βc down the positive x axis of the S frame. The relationship between coordinates in the two frames is given by:

Boost: $$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \qquad \text{and} \qquad \begin{matrix} y' = y \\ z' = z \end{matrix}$$

or: $\mathbb{X}' = O \cdot \mathbb{X}$ where: $\mathbb{X} = (\mathbf{r}, ict)$

and O is the orthogonal matrix: $$\begin{pmatrix} \gamma & 0 & 0 & i\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

1. Jack, aboard the superfast train called S', decides to make an indelible record of the length contraction he sees in the origin train station (in frame S). The train station exactly matches the train size when passengers are boarding. Plan: at $t' = 0$, he will drive two stakes into the rail bed, one from the caboose at $x' = 0$ the other from the engine at $x' = L$. Clearly because of Lorentz contraction the distance between the stakes will be larger than the station (because the station was compressed when the stakes went in). Jill sits at the rail station and watches as a highly contracted train drives stakes that are indeed spaced beyond the station. Describe (words) exactly what Jill sees.

2. Consider the event $\mathbb{X} = (3, -2, 1, i4)$. Describe (words) how you would go about finding a frame in which this event happened at the origin (i.e., $\mathbf{r}' = \mathbf{0}$). In that frame what is the time of this event? What is the speed of this frame relative to the original frame.

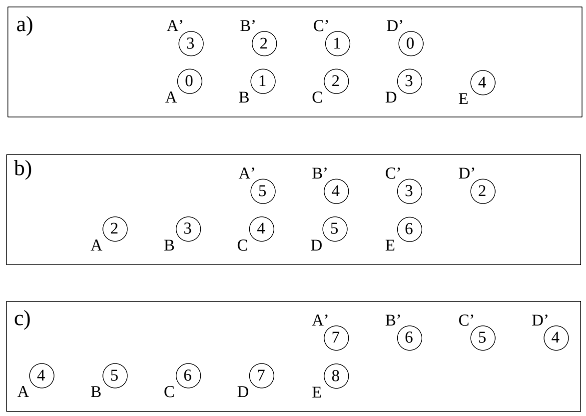
3. Two photons travel along the x-axis of S, with a constant distance L between them. Write down the location equation $x(t)$ for each photon. On the supplied Minkowski diagram draw the paths of the two photons, and label the distance between them in the S' frame. Prove in S' the distance between these photons is $L(1+\beta)^{\frac{1}{2}}/(1-\beta)^{\frac{1}{2}}$. Hint: rewrite the location equations in terms of t' and x'.

4. Consider the attached Minkowski diagram. The unit of length is light-years; the unit of time is years. Quartet, the home planet of the Quartons, is motionless in the S frame, three light years to the left of the origin. Three years ago a spaceship left Quartet (i.e., the event $(x, ct) = (-3, -3)$). According to observers in S, the spaceship traveled for 3 years toward the origin at a speed of $\frac{1}{3}c$. It then stopped and sent a radio signal back to Quartet asking if it should continue. As soon as it received the signal, Quartet replied: "yes, continue on to the origin". On the supplied diagram accurately sketch the world line of spaceship and the radio signals.

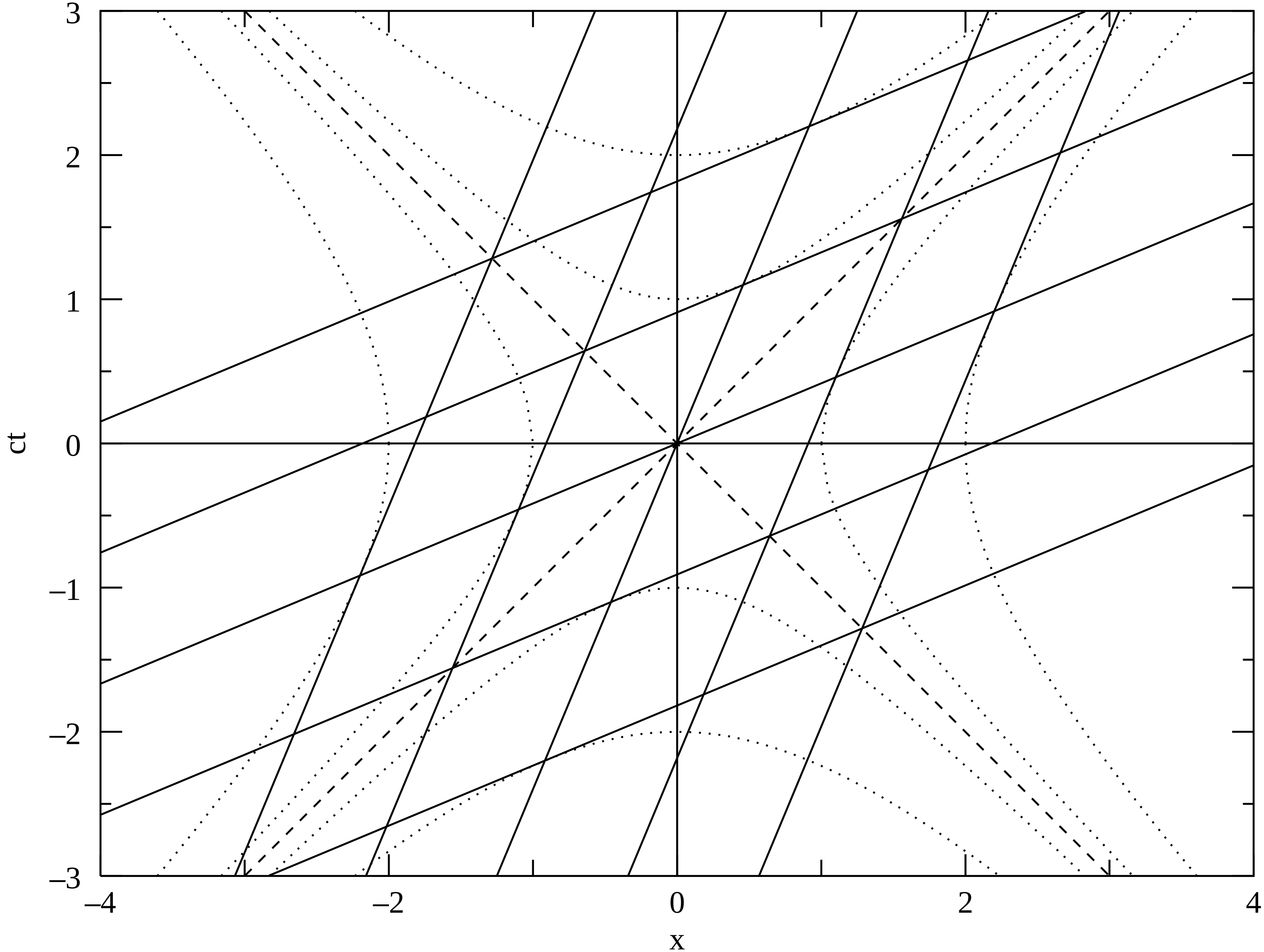
   Folks in the S frame measure the length of rod that is at rest in S': one end at $x' = 0$ the other at $x' = 1$. Label with **A** the events associated with this measurement. Show that the result is less than one light year. Folks in the S' frame measure the length of rod that is at rest in S: one end at $x = 0$ the other at $x = 1$. Label with **B** the events associated with this measurement. Show that the result is less than one light year.

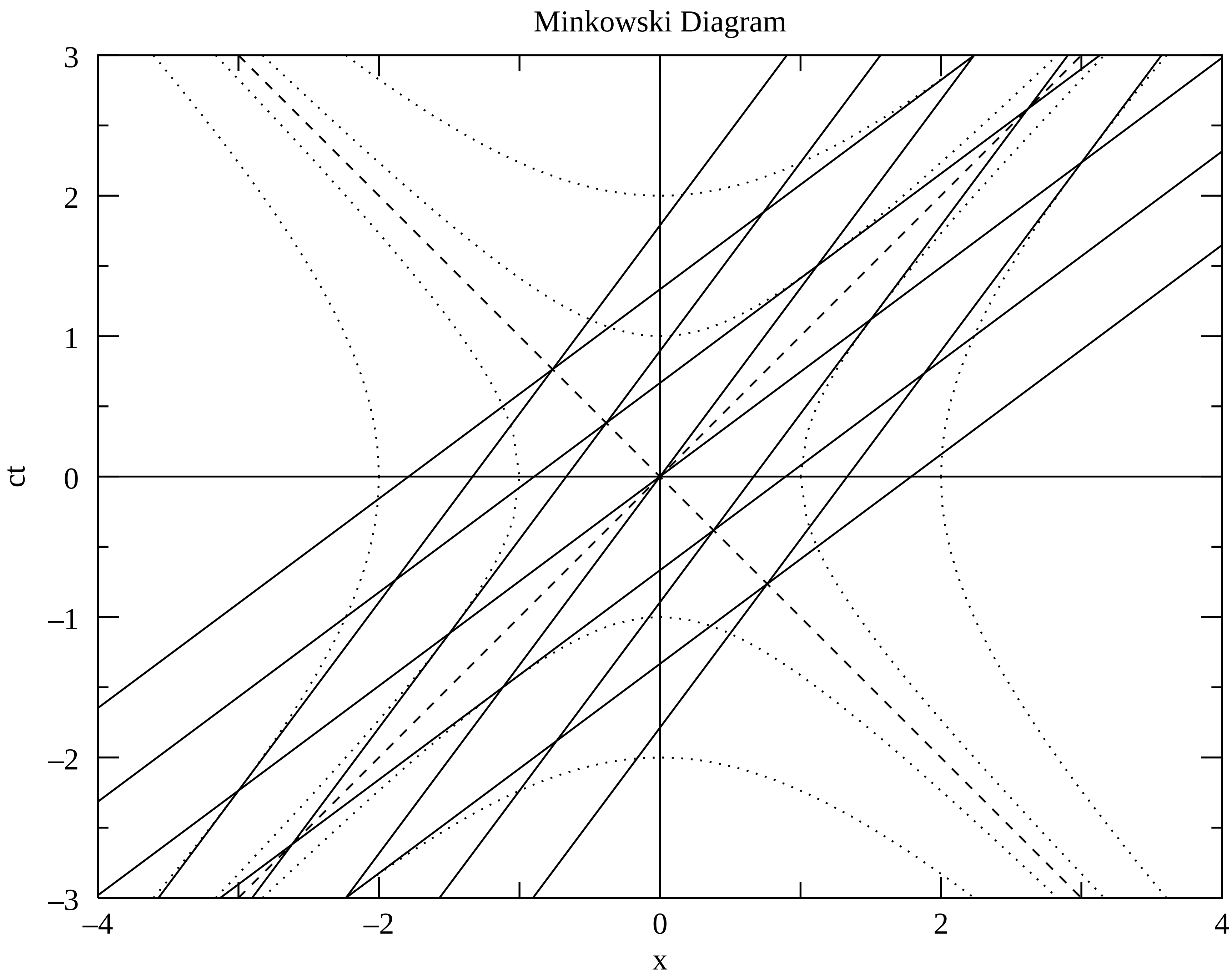
   Folks in S measure the time it takes for a calendar at $x' = 2$ to click off one year (i.e, $t' = 0 \rightarrow t' = 1$) Label with **C** the events of this measurement. Show that the result is longer than one year.

5. The diagram below shows "synchronized" clocks in frames S and S' as viewed from the CM frame (S''). Report how long it takes clock A' to click off two second as seen in S. SO what is the γ factor? Report how long it takes clock E to click off two seconds as seen in S'. As seen in S', how far apart are A and E at the time $t' = 3$? SO what is the γ factor? Call the distance between adjacent clocks as seen in the CM frame $\Delta x''$. Note that the clock A' travels a distance of $\Delta x''$ in a time $\Delta t' = 2$. Use this information to write down an equation for the velocity of the S' frame relative to the S'' frame (γ for the boost between S' and S'' should enter into this equation). Note the lack of synchronization between clocks A' and B' as seen in the CM frame: $\Delta t' = 1$ for $\Delta x''$ separation. Write down the equation describing this lack of synchronization. Solve these two equations to show that the β and γ that connect the S'' and S' frames must satisfy: $\beta^2\gamma^2 = \frac{1}{2}$. Find β.



Minkowski Diagram





Minkowski Diagram
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