The tetrahedron is the simplest of the platonic solids: four equilateral triangles combined into a pyramid structure. It can also be thought of as a cube's top diagonal and a twisted bottom diagonal. There are 24 symmetry operations that leave a tetrahedron invariant (i.e., result in a shuffling of the vertices but the the vertices remain in the same locations). Matrices are designed to perform such transformations. We examine below the symmetry operations that just involve rotations.

Find the spreadsheet `tetrahedron` from the class web site. The upper lhs contains cells to hold the values for the three Euler Angles: ϕ, θ, ψ; columns `F`–`H` contain the 3×3 rotation matrix defined by those Euler Angles. (Note use `=pi()` in the spreadsheet not some approximation of π.) The (x, y, z) location of the four vertices of the tetrahedron (labeled: A, B, C, D) occur below the matrix. Matrix multiplication results in the transformed point (x', y', z'). (Check out the spreadsheet formulas or decode the self-documentation.) A symmetry operation on the tetrahedron's vertices should just shuffle those points (i.e., each (x', y', z') should be one of the A,B,C,D inputs).

Find below the Eular Angles defining 11 rotation symmetry operations. Fill in the table showing where each vertex (A, B, C, D) is mapped by the operation. The Trace (Tr: sum of the diagonal elements) of a rotation matrix equals $1 + 2\cos\alpha$ where α is the rotation angle of the matrix. (The rotation axis direction is also encoded by the matrix but it is harder to decode.) Record the Trace of the matrix and the corresponding angle α. By seeing how the vertices are shuffled, determine yourself the rotation axis and record that also.

ϕ	θ	ψ	A	B	C	D	Tr	α	axis
π	0	0							
0	π	0							
$-\pi/2$	π	$\pi/2$							
$\pi/2$	$\pi/2$	0							
π	$\pi/2$	$\pi/2$							
$-\pi/2$	$\pi/2$	π							
0	$\pi/2$	$-\pi/2$							
$-\pi/2$	$\pi/2$	0							
π	$\pi/2$	$-\pi/2$							
$\pi/2$	$\pi/2$	π							
0	$\pi/2$	$\pi/2$